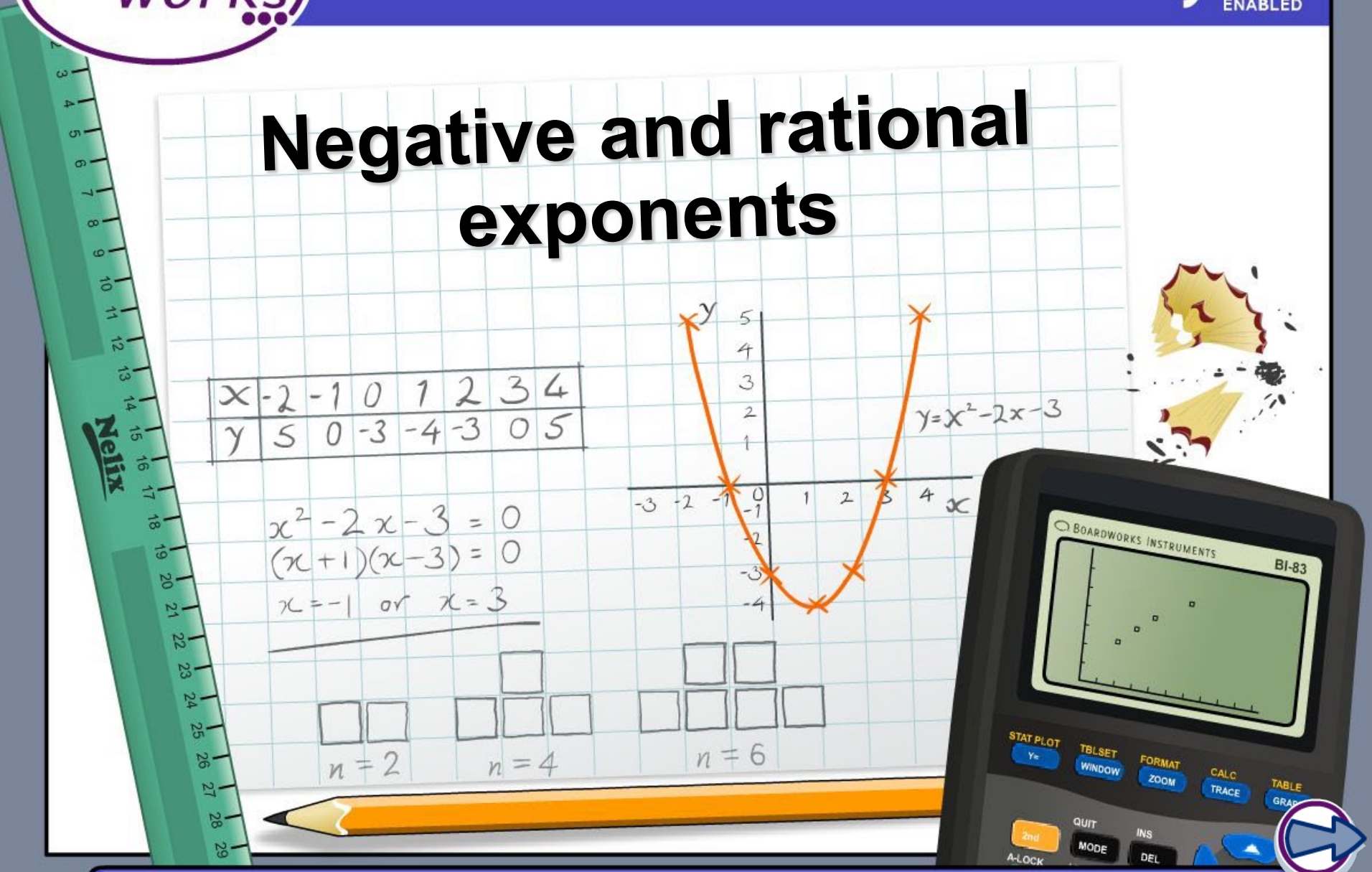
Negative and rational exponents



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Population models

The water lily population in a large pond doubles daily if left unattended. The population is measured each day, starting from the day on which there are 6 lilies.

a) Can you write an expression for the number of lilies, in terms of d, the number of days after the population was measured?

b) i) Evaluate the expression for $d = -1$, $d = 0$ and $d = 7$.

ii) What does each result represent in this situation?

Press **start** to work through the answers.

start

Negative exponents

Look at the following division:

$$b^2 \div b^4 = \frac{\cancel{b} \times \cancel{b}}{\cancel{b} \times \cancel{b} \times b \times b} = \frac{1}{b \times b} = \frac{1}{b^2}$$

Write an equivalent expression for the term $\frac{1}{b^2}$ using the division law for exponents.

Using the exponent law $x^m \div x^n = x^{(m-n)}$

$$b^2 \div b^4 = b^{(2-4)} = b^{-2}$$

This means that $b^{-2} = \frac{1}{b^2}$

In general, $x^{-n} = \frac{1}{x^n}$

Reciprocals

A nonzero number raised to the power of –1 gives us the **reciprocal** of that number.

The reciprocal of a number is what we multiply the number by to get 1.

The reciprocal of a is $\frac{1}{a}$

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

We can find reciprocals on a calculator using the x^{-1} key.

Practice questions: negative exponents

Rewrite the following expressions using fraction notation.

1.	u^{-1}	? W
2.	$2b^{-3}$	? W
3.	$y^{-2}x^{3}$	? W
4.	$2a(3-b)^{-5}$	? W

Practice questions: negative exponents

Rewrite the following expressions using negative exponents.

1.	$\frac{2}{a}$	? W
2.	$\frac{x^4}{y^3}$	? W
3.	$\frac{p^2}{q+2}$	? W
4.	$\frac{3m}{(n^2+2)^3}$	? W

Match the reciprocal pairs

Match the reciprocal pairs together

$\frac{14}{15}$	$1\frac{5}{6}$	$\frac{5}{8}$	$2\frac{1}{7}$	1.6	$1\frac{3}{7}$
0.4	$\frac{10}{17}$	$\frac{6}{11}$	0.6	$\frac{5}{11}$	$\frac{14}{19}$
$2\frac{1}{5}$	$\frac{2}{9}$	$1\frac{1}{14}$	$\frac{5}{6}$	$2\frac{1}{2}$	$1\frac{2}{3}$
1.2	$1\frac{5}{14}$	$1\frac{7}{10}$	4.5	$\frac{7}{10}$	$\frac{7}{15}$

Annual or monthly interest

Riley puts $1000 in a savings account. The first $100 does not gather interest and the rest earns interest at 3.5% per year.

Write a function for Riley's total savings after t years.
What is the equivalent monthly interest rate?
Rewrite the function using this rate.

$$f(t) = 100 + 900(1.035)^t$$

The annual interest rate, 1.035, is the monthly rate (m) multiplied by itself 12 times. This means that $1.035 = m^{12}$.

$$f(t) = 100 + 900(m^{12})^t$$

$$f(t) = 100 + 900(m)^{12t}$$

$m = 1.035^{1/12}$

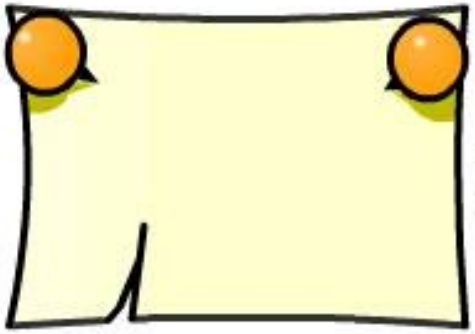
$$f(t) = 100 + 900(1.035^{1/12})^{12t}$$

The monthly rate is **0.29%**.

$$\approx 100 + 900(1.0029)^{12t}$$

Calculating exponents

Complete the calculation using your knowledge of the different exponent laws.

$35^{13} \times$  $= 35^{25}$

Exponents to use: +

Rational exponents

Suppose we have $9^{\frac{1}{2}}$. What is the value of $9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$?

Using the exponent law, $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9^1 = 9$

But also, $\sqrt{9} \times \sqrt{9} = 3 \times 3 = 9$

So in general, $x^{\frac{1}{2}} = \sqrt{x}$

The **square root** of x.

Suppose we have $8^{\frac{1}{3}}$. What is the value of $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}}$?

Using the exponent law, $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8^1 = 8$

But also, $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 2 \times 2 \times 2 = 8$

So in general, $x^{\frac{1}{3}} = \sqrt[3]{x}$

The **cube root** of x.

Rational exponents

In general, $x^{\frac{1}{n}} = \sqrt[n]{x}$

Can you find a rule for expressions in the form $x^{\frac{m}{n}}$?

We can write $x^{\frac{m}{n}}$ as $x^{\frac{1}{n} \times m}$

and we know that $(x^m)^n = x^{mn}$,

therefore, $x^{\frac{1}{n} \times m} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$

Also, we can write $x^{\frac{m}{n}}$ as $x^{m \times \frac{1}{n}}$

therefore, $x^{m \times \frac{1}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$

In general, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

Rational exponents

What is the value of $25^{\frac{3}{2}}$?

In general, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

So using the 2nd rule we can write:

$$25^{\frac{3}{2}} = (\sqrt{25})^3$$
$$= (5)^3$$
$$= \mathbf{125}$$

25 is a **perfect square** so it is easiest to use the rule where we square root first, then cube.

Notice that the numerator of the fraction determines the power and the denominator determines the degree of the root.

Calculating Exponents

Q1/5 Calculate the value of $121^{\frac{1}{2}}$.

Press the "=" button to show the work step by step.

- 121
- 14,641
- 60.5
- 11

Exponent laws summary

Link the equivalent expressions

$x^m \times x^n$	$1 / x^n$
$x^m \div x^n$	$\sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$
x^{-n}	$x^{(m-n)}$
$x^{\frac{1}{n}}$	$\sqrt{x}$
$x^{\frac{m}{n}}$	$\sqrt[n]{x}$
$x^{\frac{1}{2}}$	$x^{(m+n)}$