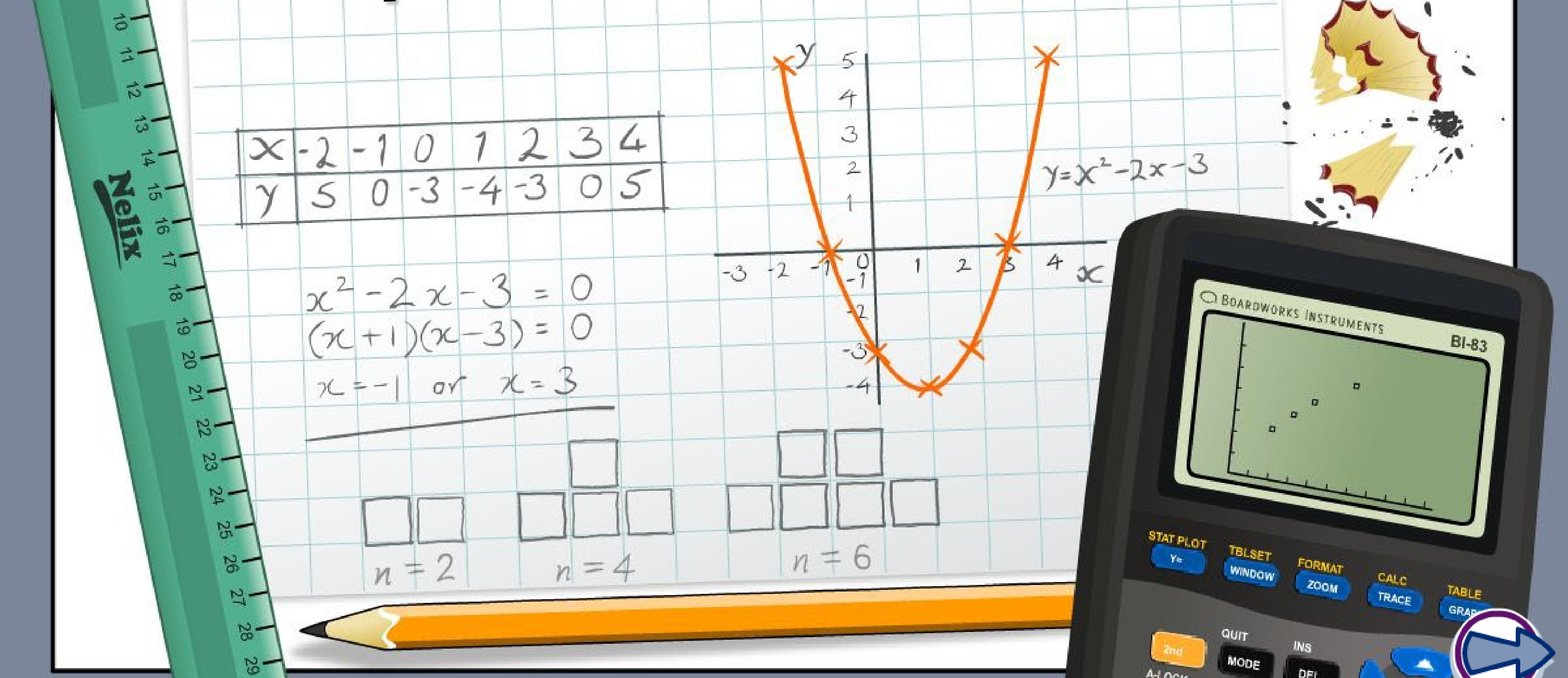
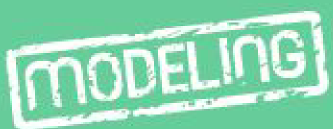
Linear, quadratic and exponential modeling



Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

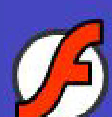
4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Interpret the function

MODELING

Read each of the descriptions below. Interpret the functions in context by writing down what each one tells you about the situation.

a) The parking fees (in \$) in a lot are modeled by the function $C(t) = 5.00 + 2.00(t - 1)$, where t is measured in hours.

b) The value (in \$) of a used car can be modeled by the function $V(t) = 9000(0.85)^t$, where t is measured in years.

c) The balance (in \$) in a savings account can be modeled by the function $A(t) = 2500(1 + 0.04/4)^{4t}$, where t is in years.

Press each of the statements to see an analysis of the function in context.

d) A large store usually sells 200 printers per month at \$90 each. It surveys its customers to assess how increasing their prices would affect sales. The money made from printer sales by applying n price increases can be modeled by the function $f(n) = (90 + 5n)(200 - 4n)$.

Comparing $y = 2x$, $y = 2^x$ and $y = x^2$

Examining a set of data for a pattern will tell us whether it fits a linear, exponential or quadratic model. Press each table to see an example.

x	y
1	2
2	4
3	6
4	8
5	10
6	12

x	y
1	2
2	4
3	8
4	16
5	32
6	64

x	y
1	1
2	4
3	9
4	16
5	25
6	36

Using the cooling formula

Newton’s Law of Cooling is: $T(t) = T_m + (T_0 - T_m)e^{-kt}$

where t = time, $T(t)$ = the temperature at time t, T_0 = the initial temperature and T_m = the temperature of the surrounding medium

Notice that this formula is a combination of a constant linear function: $y = T_m$ and an exponential function: $y = (T_0 - T_m)e^{-kt}$.

Sienna’s mom is making her an egg salad. Using an instant-read thermometer, her mom takes the hard-boiled egg when it is 205˚F and places it in 61˚F water to cool. Four minutes later the temperature of the egg is 113˚F.

Use Newton’s Law of Cooling to determine when the egg will be 68˚F, room temperature.

Using the cooling formula

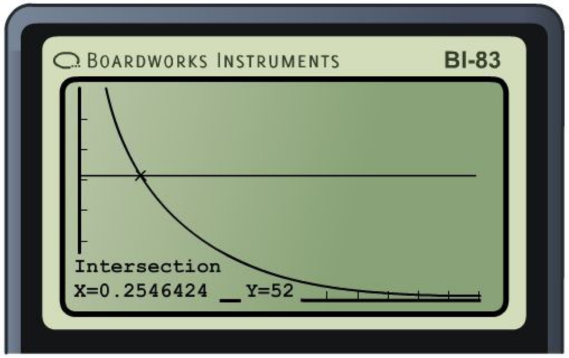
Newton's Law of Cooling is: $T(t) = T_m + (T_0 - T_m)e^{-kt}$

Substitute the known values into the formula. You can use your calculator to determine the value of 'k', then use this value to determine the time when the egg gets to 68°F.

$113 = 61 + (205 - 61)e^{-k(4)}$

$52 = 144e^{-4k}$. Solve this graphically.

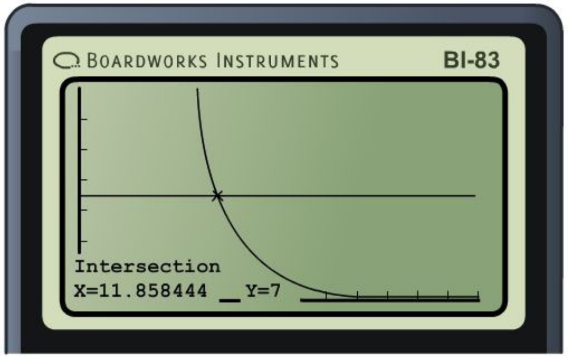
We see that $k = 0.255$.



So, when will the egg reach 68°F?

$68 = 61 + (205 - 61)e^{-0.255(t)}$

$7 = 144e^{-0.255t}$. Solve this graphically.

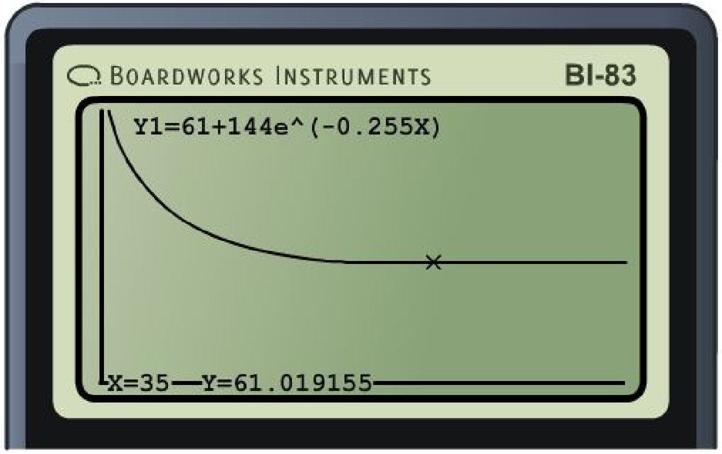


It is at 68°F at t = **11.86 minutes**.

What is the minimum temperature the egg will reach? State the range of the function modeling its temperature.

Let's graph the original function for our value of 'k', 0.255, $T(t) = 61 + (205 - 61)e^{-0.255(t)} = 61 + 144e^{-0.255t}$.

Remember the two components: $y = ab^x$ and $y = c$. We should see graphically that the egg will not cool to below a constant value: the temperature of the medium into which it is put.



This graph shows that even after t = 35 minutes, the temperature is still 61.019155°F.

The exponential part of the graph has a range of $\{y \mid y > 0\}$ and the constant part $y = 61$ makes the range $\{y \mid y > 61\}$.

Linear or exponential?

The principal of a school is looking at increases in textbook costs from two different companies over the past 7 years. Below are two tables of prices, rounded to the nearest cent.

Write a function to model each set of data.

i)

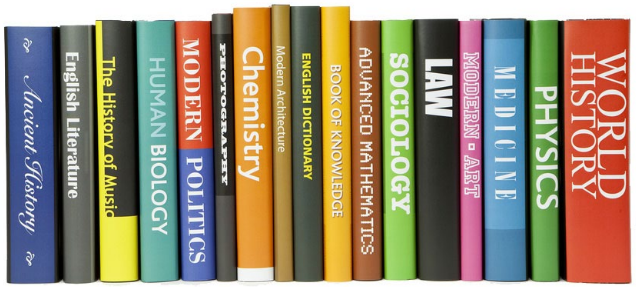
year	cost in dollars
0	$50.00
1	$52.00
2	$54.00
3	$56.00
4	$58.00
5	$60.00
6	$62.00

ii)

year	cost in dollars
0	$50.00
1	$52.00
2	$54.08
3	$56.24
4	$58.49
5	$60.83
6	$63.27

For i) the function is $y = 2x + 50$ (linear).

For ii) the function is $y = 50(1.04)^x$ (exponential).



Savings

Hey Leticia, I've saved up $900 and I'm going to keep adding $50 to it every year.

I've already got $180 more than you – of course you won't end up with more!

Well Carlota, I'm putting $720 into an account with a 7% annual interest rate. In 10 years, I'll have more than you!

Press the name of the girl who you think is right.

CARLOTA

LETICIA

Magazine price increase

Skaterz magazine needs to raise the cost of its monthly edition. The company president plans to do this gradually to avoid angering his customers. He decides to raise the cost by ½% per month for one year. It is currently selling for $3.50. Write a function to model this problem and determine the cost of the magazine in 12 months.

The cost increases each month by the same percentage, but this means that the actual increases get larger and larger. The function will be exponential:

$C(x) = 3.50(1 + 0.005)^x$, where x is in months.

After 12 monthly increases of ½%, the cost will be $3.50(1.005)^{12} = 3.71587\ldots$

= **$3.72** to the nearest cent.

Milk price increase

The cost of milk on January 1st of this year was \$4.25 per gallon and has been increasing steadily by \$0.03 per month by the end of each month.
Write a function to describe the cost and determine the cost of a gallon of milk at the end of June of this year.

The cost increases by a constant amount each month, therefore the function will be linear:

$C(x) = 4.25 + 0.03x$, where x is in months.

From the beginning of January to the end of June is 6 months so $x = 6$:

$C(6) = 4.25 + 0.03(6) = 4.43$.

At the end of June, milk costs **\$4.43**.

Depreciation

On the right is a table showing the depreciated value of machinery at a local factory over a 3 year period. The initial value of the machinery was $46,800.00.

a) Write a function to model the data.
b) Use your function to determine the machinery's value in year 10.
c) Decide whether the data is arithmetic or geometric and write a recursive formula for it.

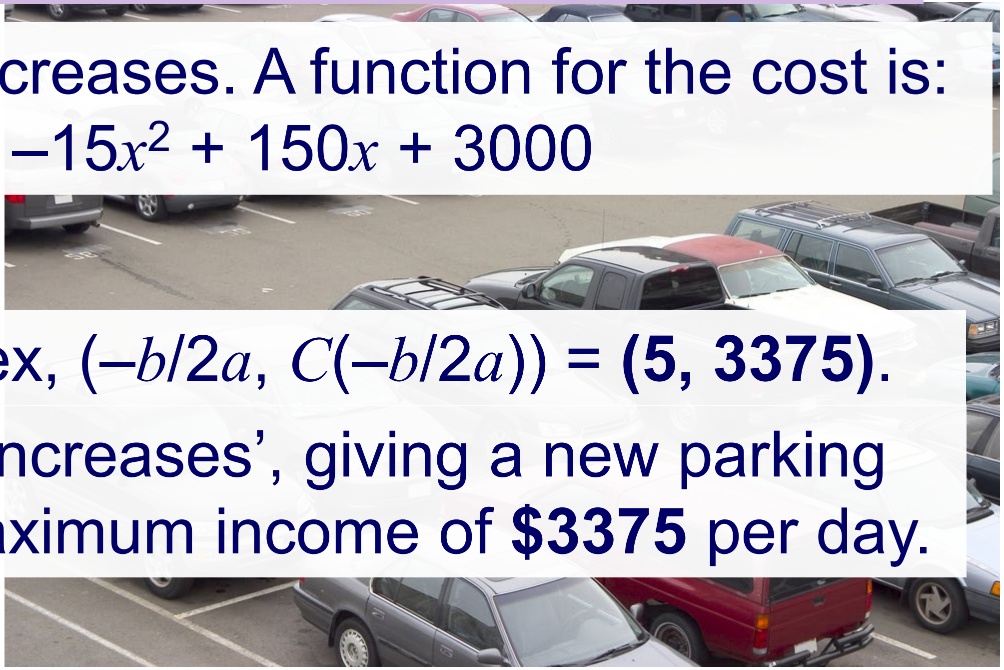
year	value of machinery
0	$46800.00
1	$42120.00
2	$37980.00
3	$34117.20

Press the **forward** arrow to see the answers to each of the questions in order.

Parking price increase

The town parking lot is raising its prices for next year. Having surveyed his customers, the manager knows he will lose 15 customers for every $1 extra he charges. They currently get 300 parkers per day and charge $10 per car.

Write a function to model the costs. What price should the town charge in order to maximize its income?



Let x be the number of $1 increases. A function for the cost is:

$C(x) = (300 - 15x)(10 + x) = -15x^2 + 150x + 3000$

This is a quadratic function.

The maximum is at the vertex, $(-b/2a, C(-b/2a))$ = **(5, 3375)**.

So there should be five '$1 increases', giving a new parking fee of **$15** and yielding a maximum income of **$3375** per day.