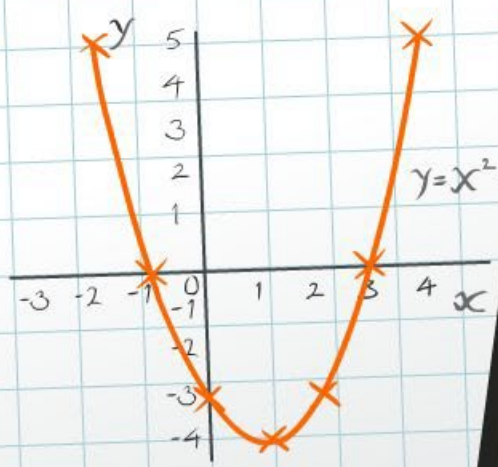
Inverse functions

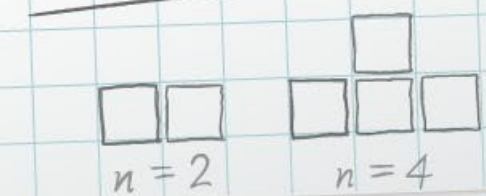
x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

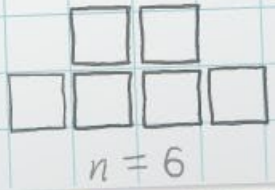
$x^2 - 2x - 3 = 0$

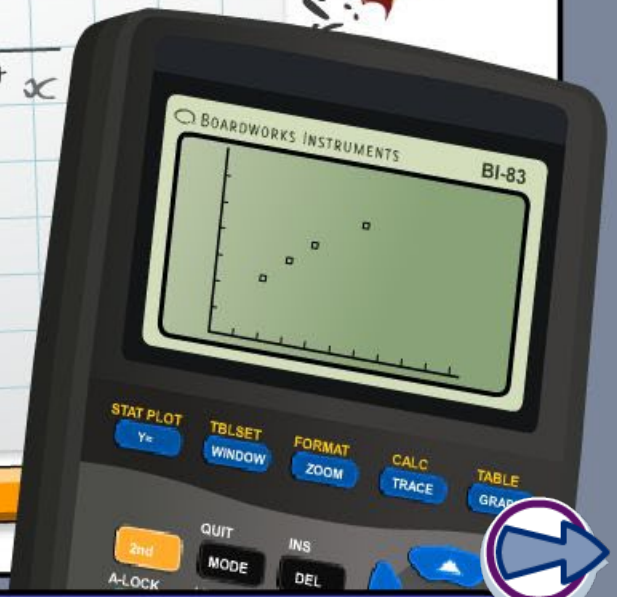
$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$









Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

What is a function?

A **function** is a combination of one or more operations that convert an **input** into an **output**.

For example, $y = 5x + 2$ is a function that converts the input, x, into the output, y, by multiplying the chosen input by 5 and adding 2.

If we choose an input $x = 6$, then $y = 5 \times 6 + 2 = \mathbf{32}$

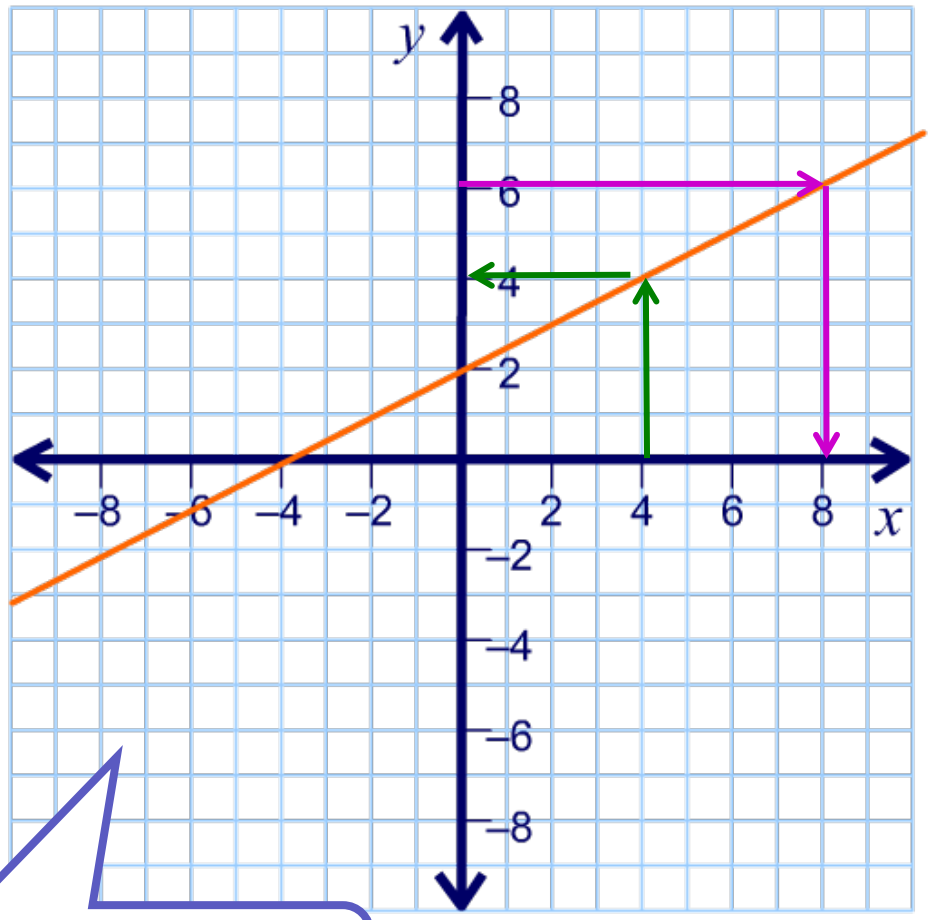
Inputs and outputs

We can use a graph to find the **output** from a given **input**. In this case, the input is an x value and the output is a y value.

For example, $x = 4$ gives $y = 4$

We can also use the same graph to find the **input** value for a given **output**.

For example, $y = 6$ is given by $x = 8$



This is the graph of $y = \frac{1}{2}x + 2$

Finding the input value

We can also find the input that gives a certain output using algebraic methods.

In the function $f(x) = 4x - 3$, what value of x gives $f(x) = 5$?

substitute 5 into the equation: $5 = 4x - 3$

rearrange: $x = \dfrac{5 + 3}{4}$

solve: $\boldsymbol{x = 2}$

Remember that x is the **input**, so it belongs to the **domain** of f.

Inverse functions

If we are not given a value for $f(x)$, we can rearrange the equation without substituting in a value:

$$f(x) = 4x - 3 \qquad \text{becomes} \qquad x = \frac{f(x) + 3}{4}$$

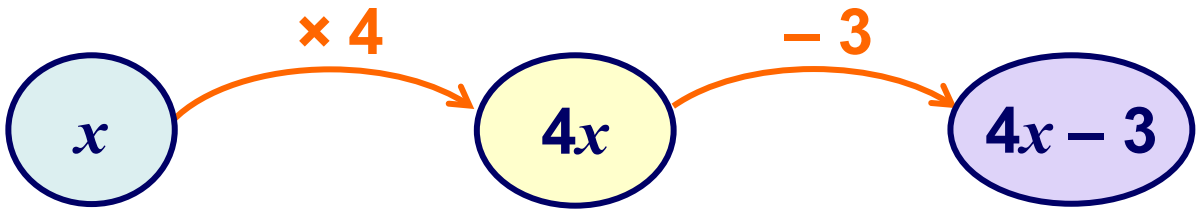
This equation gives the combination of operations that turns the output, $f(x)$, back into the input, x.

This is called the **inverse function** and is written $f^{-1}(x)$.

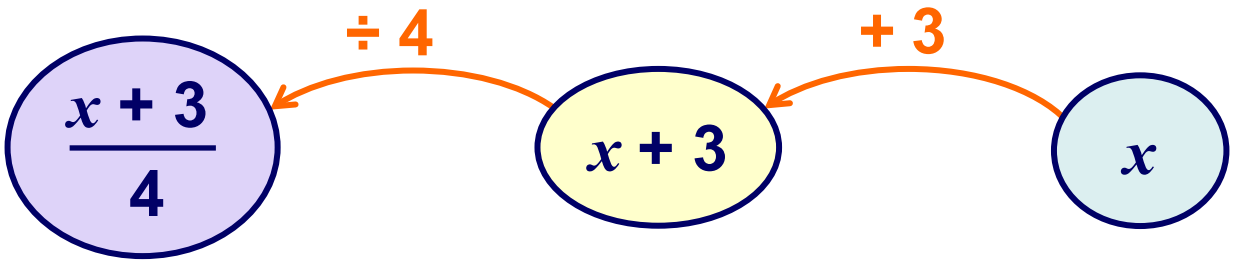
$$x = \frac{f(x) + 3}{4} \quad \Rightarrow \quad f^{-1}(x) = \frac{x + 3}{4}$$

Mapping diagrams

We can write the function $f(x) = 4x - 3$ using a **mapping diagram** showing the operations:



To find the inverse of $f(x) = 4x - 3$, we can start with x and perform the **inverse** operations in **reverse** order.



And so for $f(x) = 4x - 3$, $f^{-1}(x) = \dfrac{x + 3}{4}$.

Finding the inverse function

To find the inverse of $f(x)$, rearrange to make x the subject.

old input, x → **becomes** → **new output**

old output, y → **becomes** → **new input**

What is the inverse of $y = 3x + 2$?

subtract 2 from both sides: $y - 2 = 3x$

divide both sides by 3: $\frac{y-2}{3} = x$

write so that x is on the left: $x = \frac{y-2}{3}$

$$f^{-1}(y) = \frac{y-2}{3}$$

Check your answer

If we apply $f(x)$ to a number and then apply $f^{-1}(x)$ to the result, we should get back to the original number.

We can use this to check our results:

Let $x = 2$.

$f(x) = 3x + 2$

$f(2) = (3 \times 2) - 8$

$= 6 - 8$

$f(2) = -2 = y$

$f^{-1}(y) = \dfrac{y - 2}{3}$

$f^{-1}(-2) = \dfrac{(-2) + 8}{3}$

$= \dfrac{6}{3}$

$f^{-1}(-2) = 2 = x$

Match functions with their inverses

Match these functions to their inverses

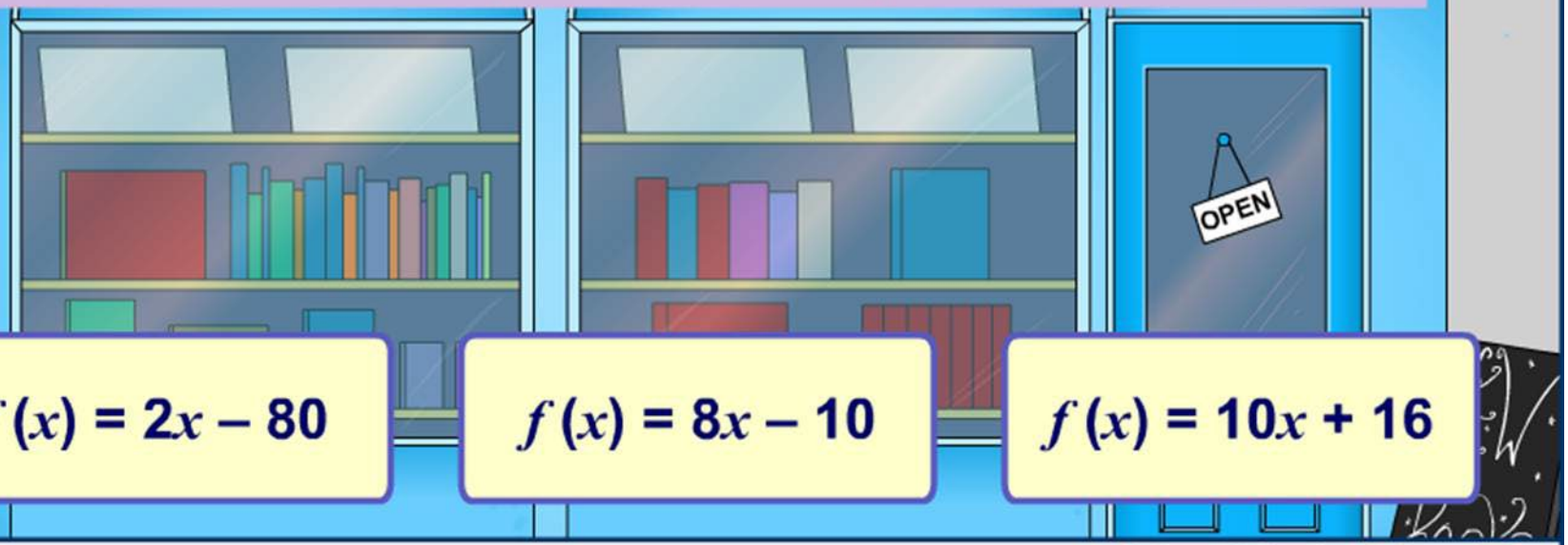
$f(x) = \frac{x+2}{5}$	$f^{-1}(x) = x - 5$
$f(x) = 2 - x$	$f^{-1}(x) = 5x - 2$
$f(x) = x + 5$	$f^{-1}(x) = \frac{x-5}{2}$
$f(x) = 2x + 5$	$f^{-1}(x) = 2 - x$

Inverse function basketball

Bookstore functions

Aisha runs a discount bookstore where all books cost $2.
She hires one shop clerk at $10 an hour for 8 hours each day.

If x is the number of books sold in a day, which function gives Aisha's profit each day?



$f(x) = 2x - 80$

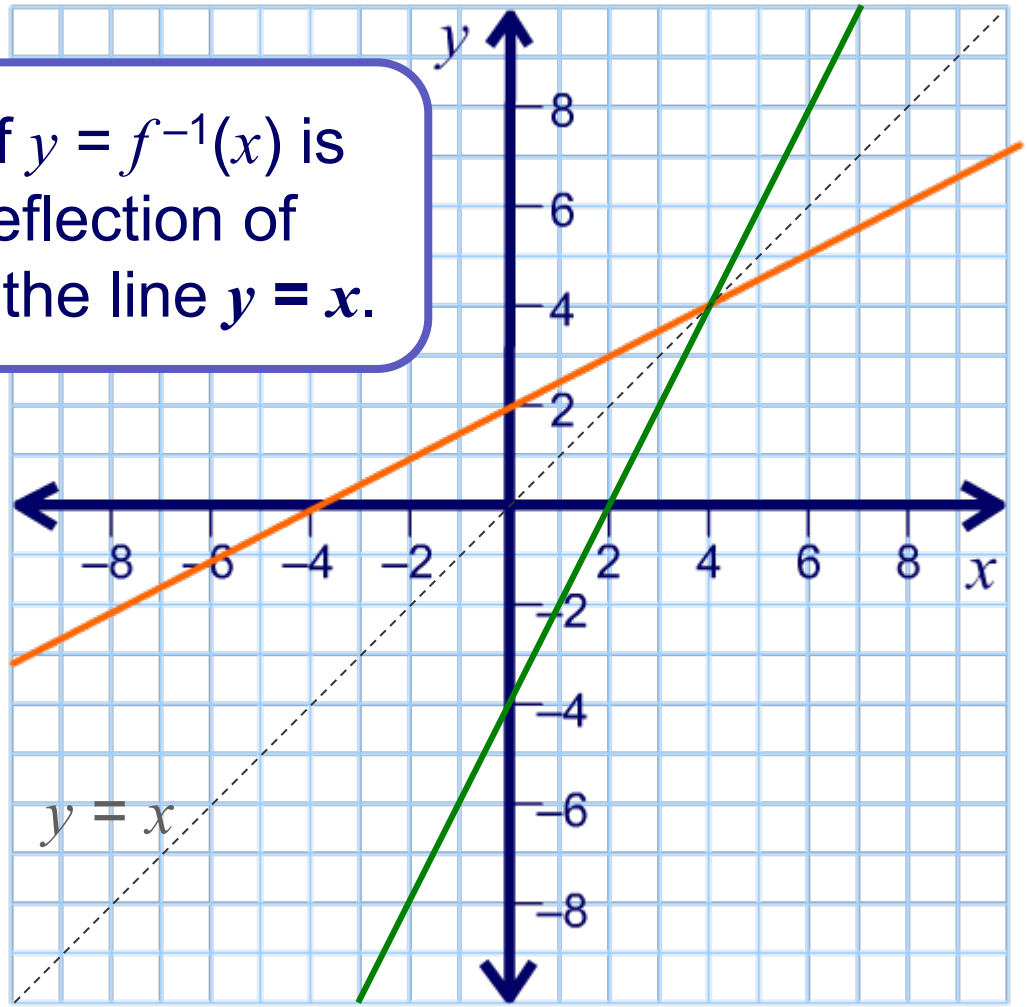
$f(x) = 8x - 10$

$f(x) = 10x + 16$

Graphing inverse functions

Remember that the **domain** of $f^{-1}(x)$ is the **range** of $f(x)$, and the **range** of $f^{-1}(x)$ is the **domain** of $f(x)$.

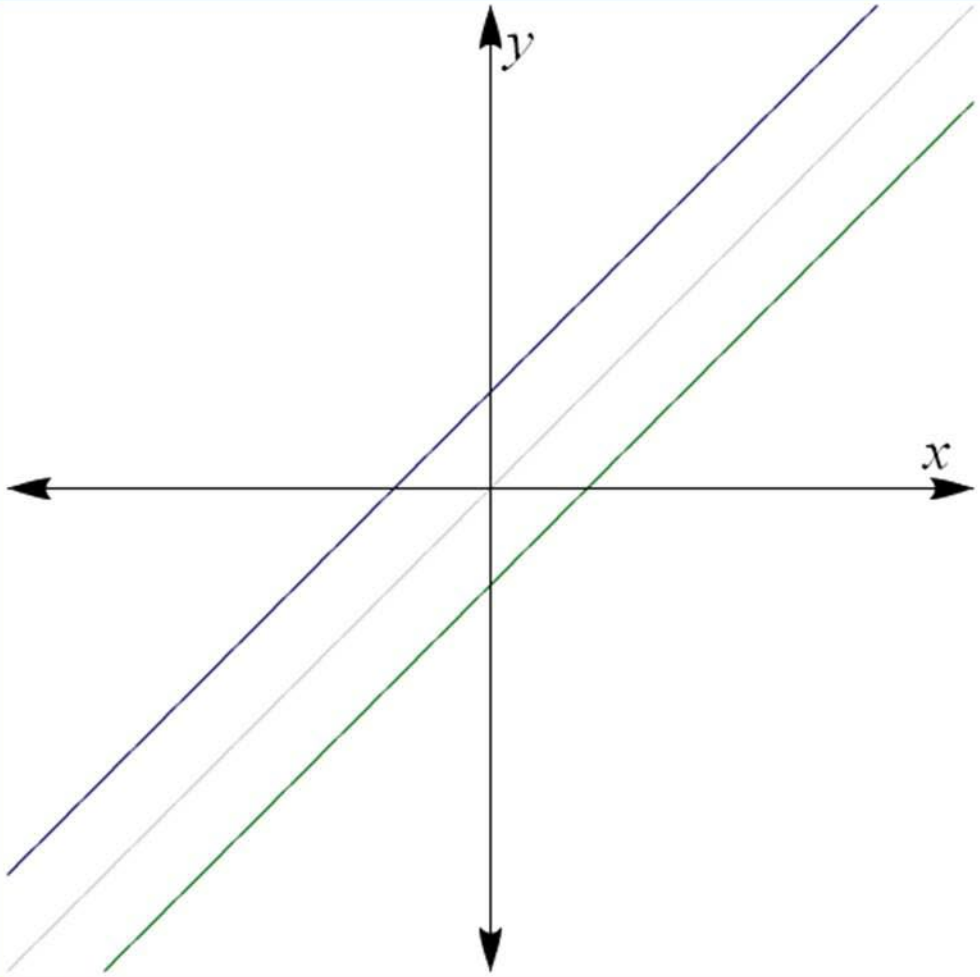
The graph of $y = f^{-1}(x)$ is always a reflection of $y = f(x)$ about the line $y = x$.



$y = \frac{1}{2}x + 2$

$y = 2x - 4$

Graphs of inverse functions



$f(x) = x + 1$

$f^{-1}(x) = x - 1$