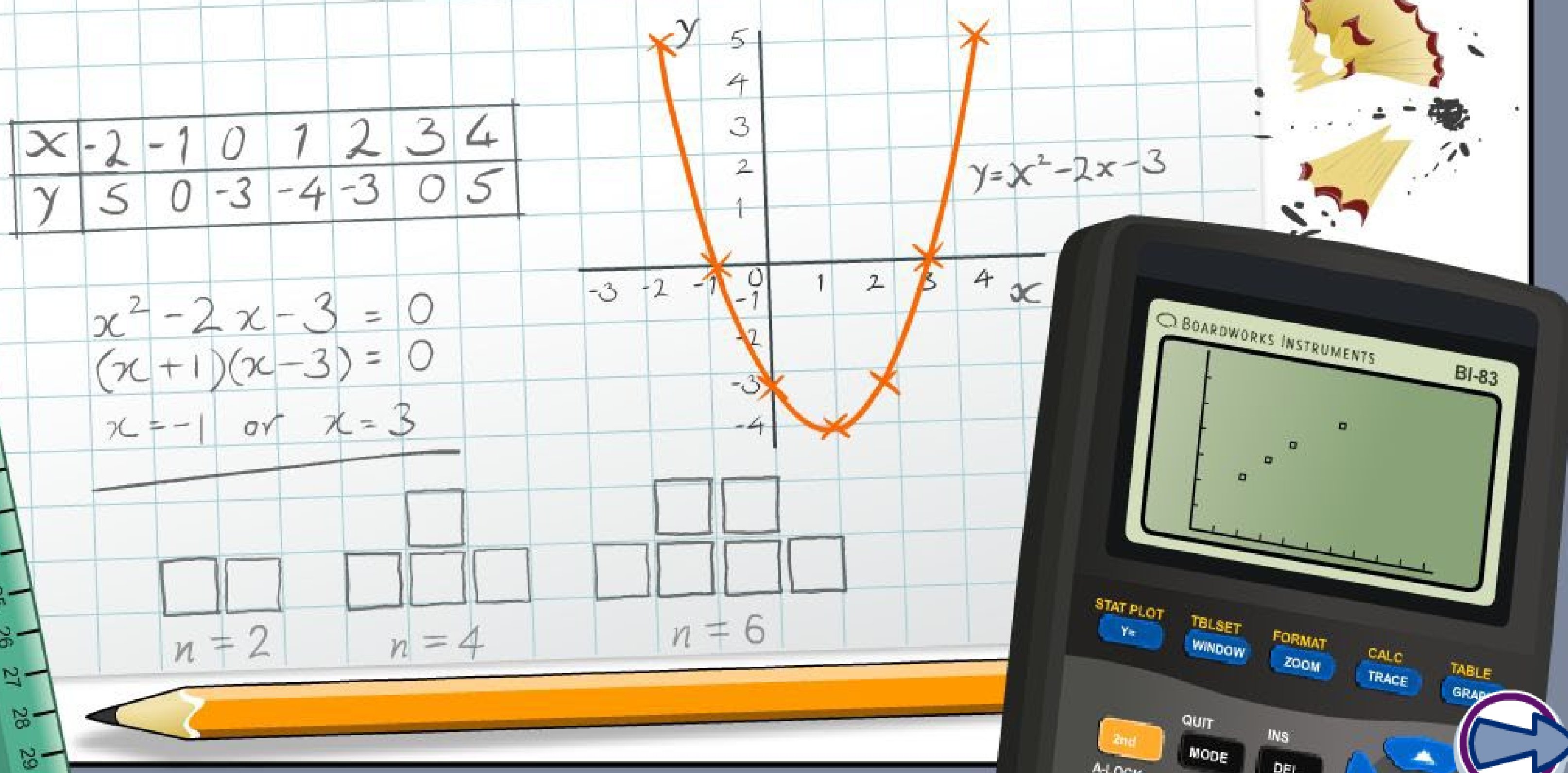
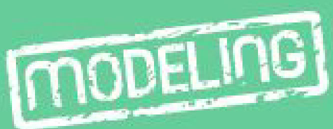
Geometric sequences



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

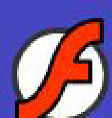
4) Model with mathematics.

5) Use appropriate tools strategically.

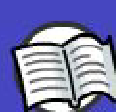
6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

Geometric sequences

What do these sequences have in common?

8, 16, 32, 64, 128 … 1, 3, 9, 27, 81… 20, 10, 5, 2.5, 1.25…

multiply by 2 **multiply by 3** **multiply by ½**

–4, –20, –100, –500, –2500… 1, –2, 4, –8, 16…

multiply by 5 **multiply by –2**

All of these sequences are made by **multiplying** each term by the same number to get the next term. They are **geometric sequences**.

The number that each term is multiplied by to get the next term is called the **common ratio, *r*,** of the sequence.

Sequences that decrease by dividing

Can you figure out the next three terms in this sequence?

$$1024, \quad 256, \quad 64, \quad 16, \quad 4, \quad 1, \quad \frac{1}{4}, \quad \frac{1}{16}, \quad \ldots$$

÷4 ÷4 ÷4 ÷4 ÷4 ÷4 ÷4

How did you figure these out?

This sequence starts with 1024 and decreases by dividing by 4 each time.

Dividing by 4 is equivalent to multiplying each term by ¼ to give the next term.

Recursive definition

Each term in a sequence is identified by its position in the sequence.

The first term is a_1, because it is in position 1.

The term in position n, where n is a natural number, is called a_n.

A geometric sequence can be defined **recursively** by the formula:

$$a_{n+1} = ra_n$$

where r is the common ratio of the sequence.

The value of the first term needs to be given as well, so that the definition is unique.

Finding the ratio

The common ratio r of a geometric sequence can be found by dividing any term in the sequence by the one before it.

Find the common ratio for this geometric sequence:

8, 12, 18, 27, 40.5, …

Choose any term and divide it by the one before it:

$$r = 12 \div 8$$

$$= 1.5$$

The sequence continues by multiplying the previous term by 1.5.

Find the next two terms in this geometric sequence:

2000	200	20		

Practice with recursive definitions

Match the recursive definition to the correct sequence.

	5, 10, 20, 40, 80...
	1, –5, 25, –125, 625...
	–1, –10, –100, –1000...
	13, 52, 208, 832, 3328...
	7, 21, 63, 189, 567,...

$a_{n+1} = -5a_n$

S C ?

The general term

How do you find the **general term** of a geometric sequence?

Find the nth term of the sequence, 3, 6, 12, 24, 48, …

$$3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \xrightarrow{\times 2} 24 \xrightarrow{\times 2} 48 \ldots$$

This is a geometric sequence with 2 as the common ratio.
We could write this sequence as:

$$3,\quad 3 \times 2,\quad 3 \times 2 \times 2,\quad 3 \times 2 \times 2 \times 2,\ \ldots\quad 3 \times \underbrace{2 \times \ldots \times 2}_{n - 1 \text{ times}}$$

or

$$3,\quad 3 \times 2,\quad 3 \times 2^2,\quad 3 \times 2^3,\ \ldots\quad 3 \times 2^{(n-1)}$$

The general term of this sequence is $\mathbf{3 \times 2^{(n-1)}}$.

Explicit formula

If we call the first term of a geometric sequence a_1 and the common ratio r, then we can write a general geometric sequence as:

1st	2nd	3rd	4th	5th	
a_1,	$a_1 \times r$,	$a_1 \times r^2$,	$a_1 \times r^3$,	$a_1 \times r^4$,	…

$\times r$ $\times r$ $\times r$ $\times r$

The **explicit formula** for the geometric sequence with first term a_1 and common ratio r is given by

$$\boxed{a_n = a_1 r^{n-1}}$$

Practice with explicit formulas

Match the explicit formula to the correct sequence.

2, 4, 8, 16, 32...

–1, –4, –16, –64, –256...

9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$...

3, 6, 12, 24, 48 ...

1, 7, 49, 343, 2401...

$a_n = -(4^{n-1})$

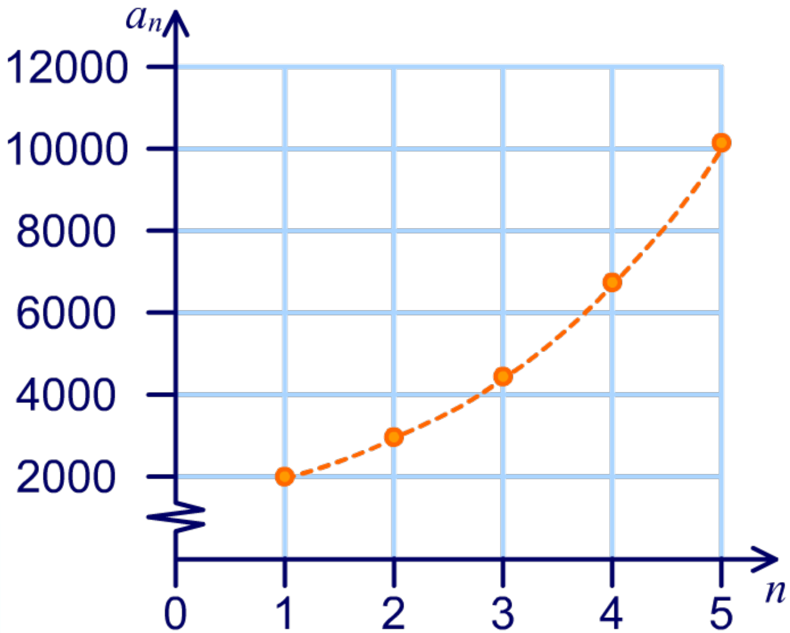
Graphing a geometric sequence

The population in Boomtown over 5 years is given in the table.

n	1	2	3	4	5
year	2006	2007	2008	2009	2010
population	2,000	3,000	4,500	6,750	10,125

This can be modeled by a geometric sequence with explicit formula $a_n = 2000 \times 1.5^{n-1}$.

We can plot the data as points on a graph with n along the x-axis and a_n along the y-axis.



The points lie along an **exponential** function.

Investments

Geometric sequences often occur in real life situations where there is a repeated percentage change.

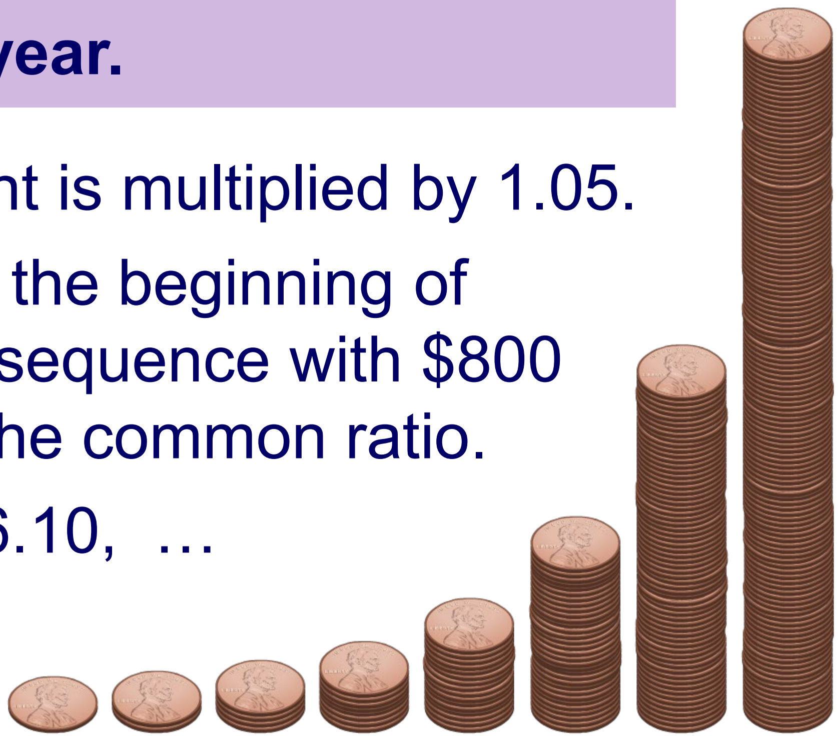
For example, \$800 is invested in an account with an annual interest rate of 5%.

Write a formula for the value of the investment at the beginning of the n^{th} year.

Step 1) Every year the amount is multiplied by 1.05.

The amount in the account at the beginning of each year forms a geometric sequence with \$800 as the first term and 1.05 as the common ratio.

$$\$800 \xrightarrow{\times 1.05} \$840 \xrightarrow{\times 1.05} \$882 \xrightarrow{\times 1.05} \$926.10, \ \ldots$$



Investments

When \$800 is invested in an account with an annual interest rate of 5%, the amount in the account at the beginning of each year forms a geometric sequence with \$800 as the first term and 1.05 as the common ratio.

Step 2) We can write the sequence as:

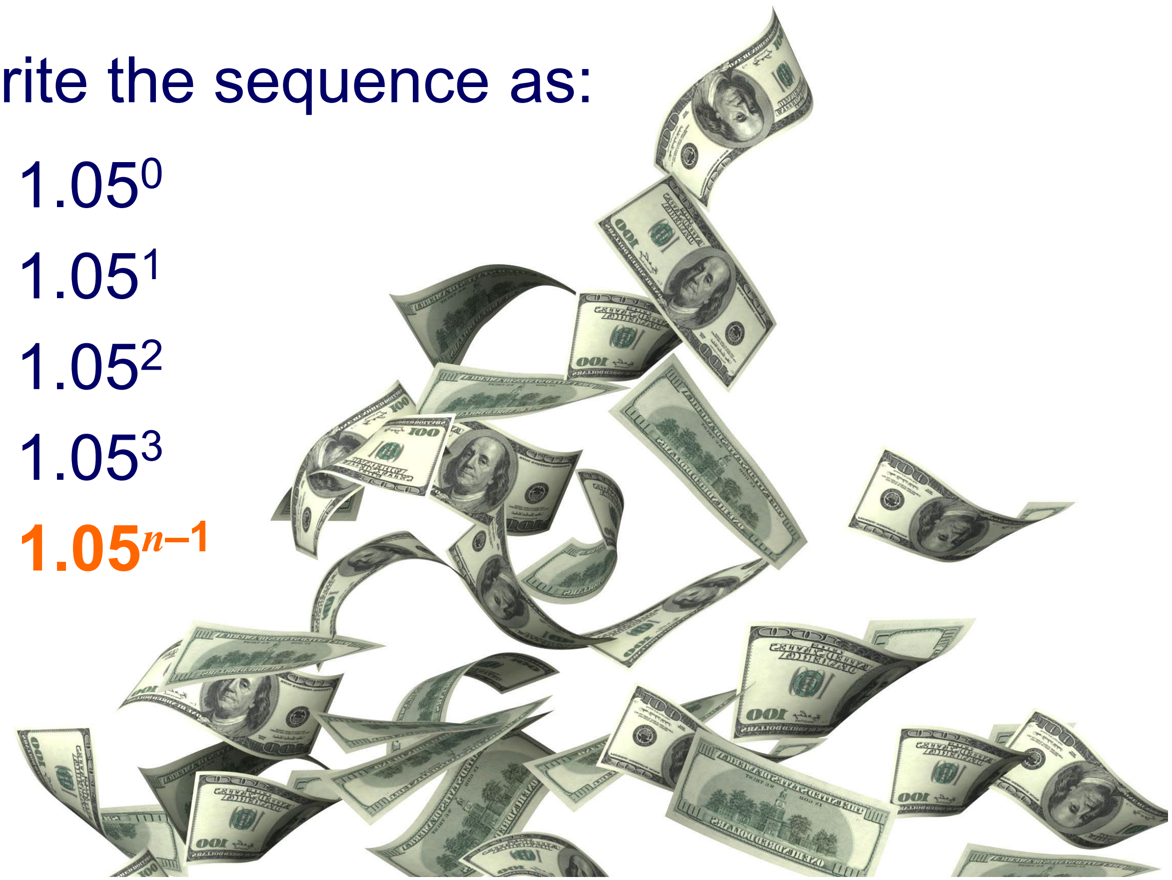
$a_1 = \$800 \times 1.05^0$

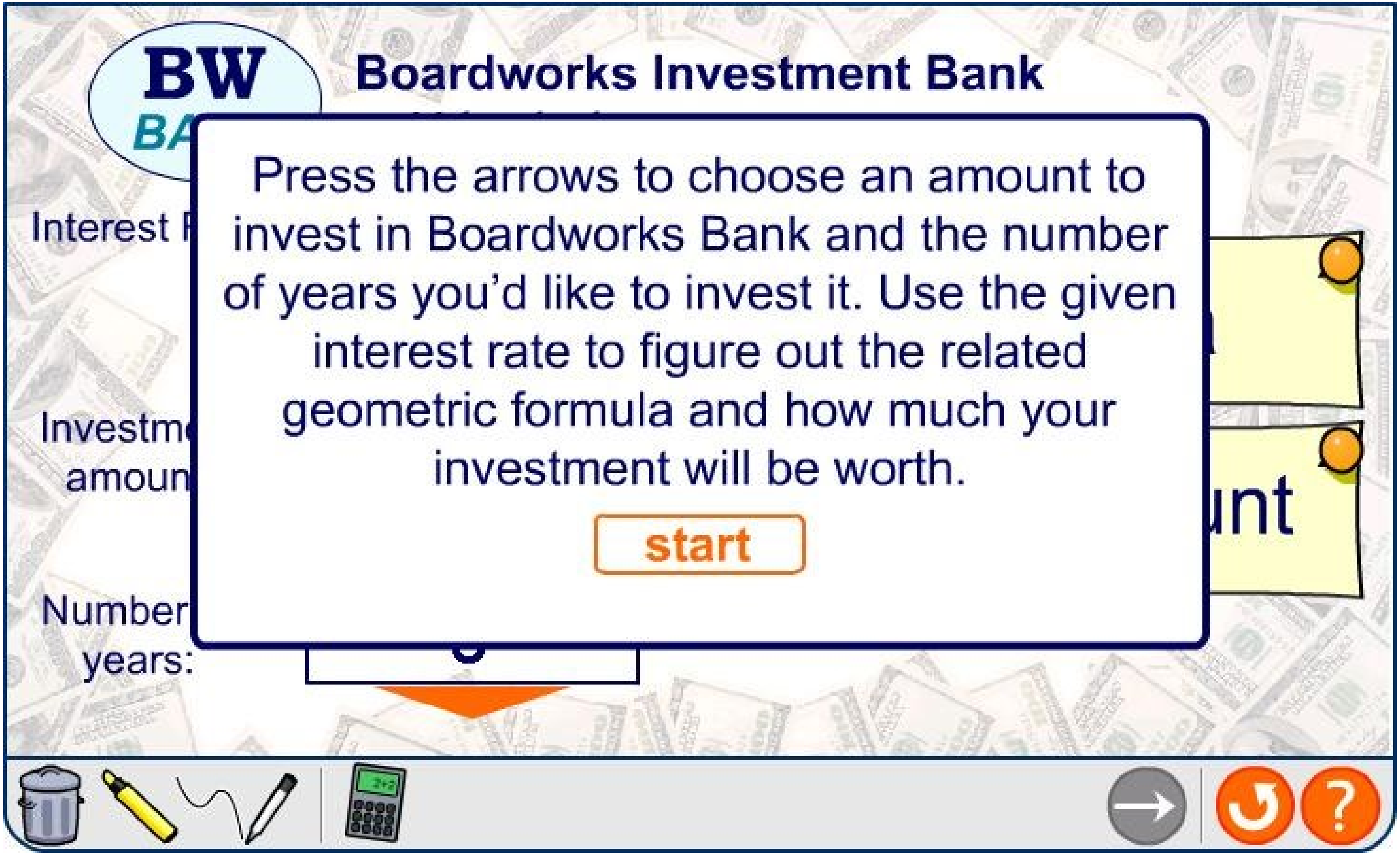
$a_2 = \$800 \times 1.05^1$

$a_3 = \$800 \times 1.05^2$

$a_4 = \$800 \times 1.05^3$

$\boldsymbol{a_n = \$800 \times 1.05^{n-1}}$




Boardworks Investment Bank
Press the arrows to choose an amount to invest in Boardworks Bank and the number of years you'd like to invest it. Use the given interest rate to figure out the related geometric formula and how much your investment will be worth.
start