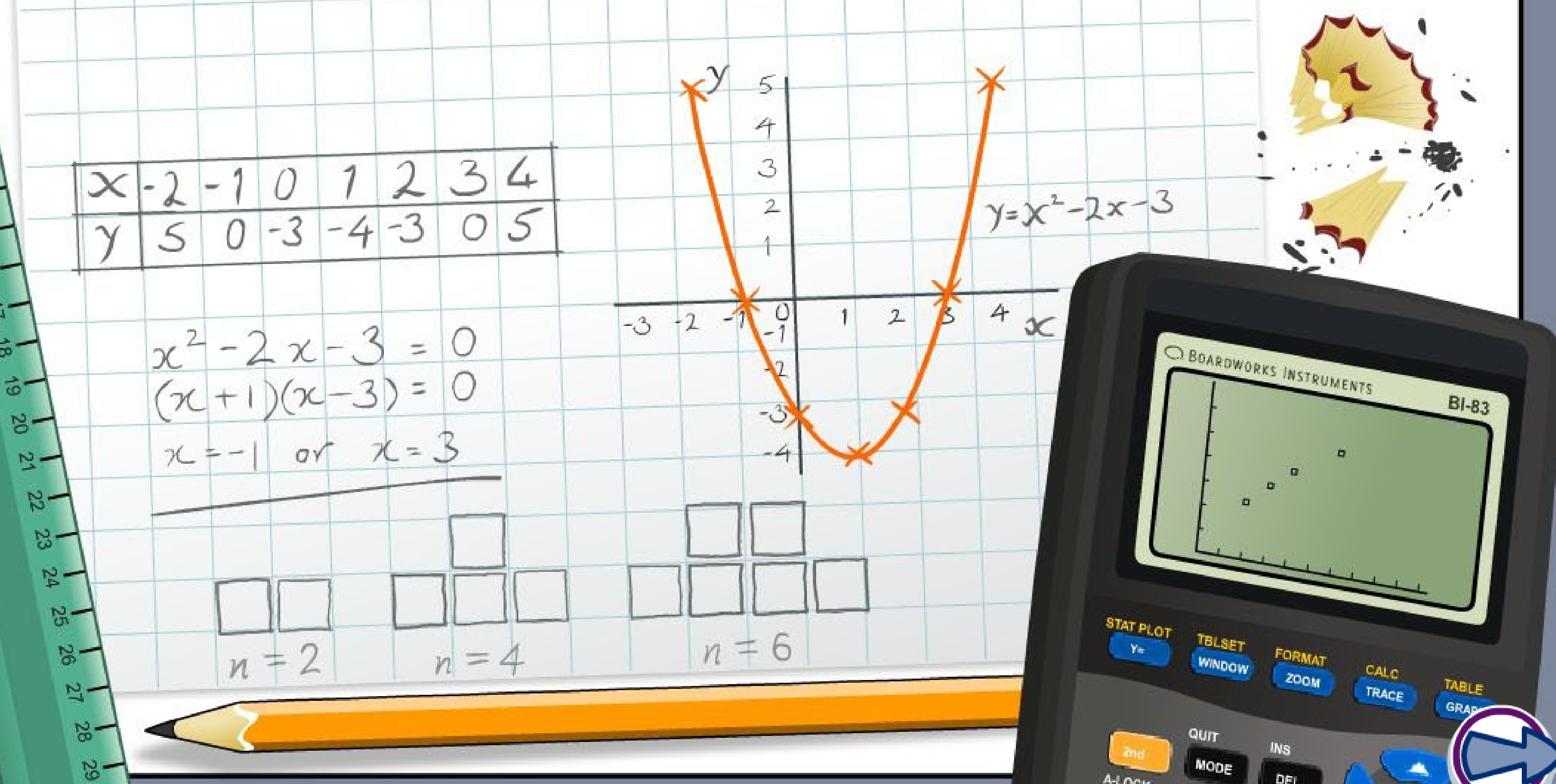
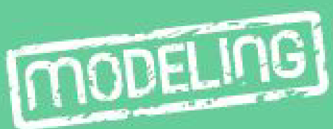
Factoring quadratics



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.



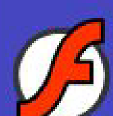
Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) **Make sense of problems and persevere in solving them.**
2) **Reason abstractly and quantitatively.**
3) **Construct viable arguments and critique the reasoning of others.**
4) **Model with mathematics.**
5) **Use appropriate tools strategically.**
6) **Attend to precision.**
7) **Look for and make use of structure.**
8) **Look for and express regularity in repeated reasoning.**



This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Quadratic expressions

A **quadratic expression** is an expression in which the highest power of the variable is 2.

For example, $x^2 - 2$, $w^2 + 3w + 1$, $4 - 5g^2$, $\frac{t^2}{2}$

The general form of a quadratic expression in x is:

$$ax^2 + bx + c \qquad \text{(where } a \neq 0\text{)}$$

x is a **variable**.

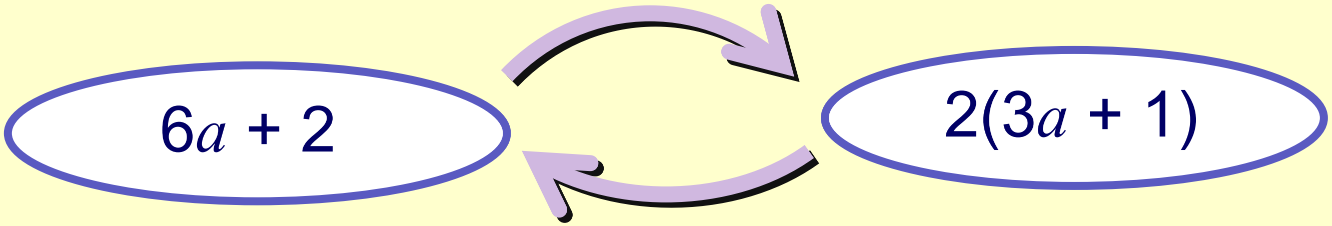
a is a fixed number and is the **coefficient** of x^2.

b is a fixed number and is the **coefficient** of x.

c is a fixed number and is the **constant term**.

Factoring an expression is the opposite of expanding it.

expanding (or using the **distributive law**)



factoring

How would you factor the expression $x^2 + 6x$?

Both terms in this expression share a common factor of x.

We can therefore write:

$$x^2 + 6x = \mathbf{x(x + 6)}$$

Factoring quadratic expressions

Quadratic expressions of the form $x^2 + bx + c$ can be factored if they can be written using parentheses as

$$(x + d)(x + e)$$

where d and e are integers.

If we multiply $(x + d)(x + e)$ we have:

$$(x + d)(x + e) = x^2 + dx + ex + de$$
$$= x^2 + (d + e)x + de$$

Comparing this to $x^2 + bx + c$ we can see that:

- The sum of d and e must equal b, the coefficient of x.
- The product of d and e must equal c, the constant term.

Factoring quadratics: an example

How would you factor the expression: $x^2 + 9x + 20$?

We need two numbers that add to make +9 and multiply to make +20.

First, look at the numbers that multiply to make +20:

1 × 20	–1 × –20
2 × 10	–2 × –10
4 × 5	–4 × –5

We need to pick the pair that sum to +9, which is +4 and +5.

The answer must therefore be: $(x + 4)(x + 5)$

Can you correctly factor these expressions?

$$x^2 - 10x + 24 = (x \quad)(x \quad)$$

Select pairs of equivalent expressions to match them.

$(x + 4)(x - 8)$	$(x - 4)(x - 1)$	$x^2 - 5x + 4$
$(x - 4)(x - 4)$	$x^2 - 12x + 27$	$x^2 + 2x - 48$
$(x + 2)(x + 8)$	$(x + 8)(x - 6)$	$x^2 - 8x + 16$
$x^2 + 10x + 16$	$x^2 - 4x - 32$	$(x - 3)(x - 9)$

When the coefficient of x^2 is not 1

Quadratic expressions of the form $ax^2 + bx + c$ can be factored if they can be written using parentheses as

$$(dx + e)(fx + g)$$

where d, e, f and g are integers.

If we multiply $(dx + e)(fx + g)$ we have,

$$(dx + e)(fx + g) = dfx^2 + dgx + efx + eg$$
$$= dfx^2 + (dg + ef)x + eg$$

Comparing this to $ax^2 + bx + c$ we can see that we must choose d, e, f and g such that:

$$a = df$$
$$b = (dg + ef)$$
$$c = eg$$

Where do I start?

To factor $ax^2 + bx + c$ as $(dx + e)(fx + g)$, choose d, e, f and g such that: $a = df$, $b = (dg + ef)$ and $c = eg$

It is difficult to identify these number immediately, so follow these steps:

- Find the factor pairs of a. These are possibilities for d and f (if a is prime, d and f can only equal a and 1).
- Find the factor pairs of c. These are possibilities for e and g.
- Identify the one combination of these factors such that when the parentheses are multiplied out, the terms containing x sum to give bx, so that $(dg + ef) = b$.

$$(dx + e)(fx + g) \qquad dgx + efx = bx$$

How would you go about trying to factor $2x^2 + 9x + 7$?

Factor it by looking at what we can be certain of and building up from there.

$$(2x + 7)(x + 1)$$

- The $2x^2$ means that there is only one possibility for what must go before the xs, which is 2 and 1.
- Since all the terms are positive, there must be a + sign in both brackets.
- Since 7 can only be 7 × 1, this is the only possible pair for the remaining numbers. When multiplied out, the x terms need to sum to $+9x$, so we need to make sure they go in the correct set of parentheses. Since $(2x \times 1) + (x \times 7) = 9x$ we know that the 7 must go in the left set of parentheses.

Another example

How would you factor the expression: $6x^2 + 9x – 6$?

$$6x^2 + 9x - 6$$
$$= 3(2x^2 + 3x - 2)$$
$$= 3(2x - 1)(x + 2)$$

If there is a common factor, always take this out first.

- Take out the common factor 3.
- Since the coefficient of x^2 in the bracket is 2, we know the factored version will have $2x$ and x.
- The factors of –2 are: –2 and 1 or 2 and –1. When multiplied out, the x terms must sum to $3x$. Notice that $(2 \times 2x) + (-1 \times x)$ is $3x$, so –1 must go in the left set of parentheses and 2 must go in the right set.

Check the answer by expanding again.

Can you correctly factor these expressions?

$$25x^2 - 25x + 6 = (\square x - \square)(\square x - \square)$$

Matching quadratic expressions 2

Select pairs of equivalent expressions to match them.

$2x^2 + 13x - 45$	$7x^2 + 67x + 36$	$(x + 3)(x + 6)$
$(7x + 4)(x + 9)$	$(x - 8)(5x + 4)$	$(x + 9)(2x - 5)$
$(4x + 2)(x + 9)$	$(x + 1)(2x + 7)$	$5x^2 - 36x - 32$
$x^2 + 9x + 18$	$2x^2 + 9x + 7$	$4x^2 + 38x + 18$

Factoring the difference of squares

A quadratic expression in the form

$$x^2 - y^2$$

is called the **difference of squares**.

The difference of squares can be factored as follows:

$$x^2 - y^2 = (x + y)(x - y)$$

For example,

$9x^2 - 16 = (3x)^2 - 4^2 = \mathbf{(3x + 4)(3x - 4)}$

$25a^2 - 1 = (5a)^2 - 1^2 = \mathbf{(5a + 1)(5a - 1)}$

$m^4 - 49n^2 = (m^2)^2 - (7n)^2 = \mathbf{(m^2 + 7n)(m^2 - 7n)}$

Factor these expressions using the difference of squares.

$$64x^2 - 9y^2 = (\square + \square)(\square - \square)$$

The difference of squares

Select pairs of equivalent expressions to match them.

$81x^2 - 36$	$x^2 - 81$	$(3x + 4)(3x - 4)$
$49x^2 - 25$	$9x^2 - 16$	$(7x + 9)(7x - 9)$
$36x^2 - 4$	$(6x + 2)(6x - 2)$	$(9x + 6)(9x - 6)$
$(7x - 5)(7x + 5)$	$49x^2 - 81$	$(x + 9)(x - 9)$

Proofs using factoring

The following questions involve proving results using factoring. Discuss them either in groups or together as a class.

Press **start** to begin.

start

Find the height of the painting

The width of a painting is 4 cm more than its height. The area of the painting is 45 cm². Find its height.

If we call the height of the painting x, we can label it like this:

Using the information about the area of the painting, we can write an equation:

height × width = area

$$x(x + 4) = 45$$

Find the height of the painting

The solution to $x(x + 4) = 45$ will give us the height of the painting.

In this example, we can see that $x = 5$ cm is a possible solution to this equation, because

$$5(5 + 4) = 5 \times 9 = 45\,\text{cm}$$

The height of the rectangle is therefore **5 cm**.

However, there is another value of x that will also solve the equation $x(x + 4) = 45$. This is $x = -9$.

This is because $x(x + 4) = 45$ is an example of a **quadratic equation**. These usually have two solutions.

Solving quadratic equations

The general form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

where a, b and c are constants and $a \neq 0$.

We usually arrange quadratic equations like this so that all the terms are on the left-hand side of the equals sign, as it makes them easier to solve.

Rearranging the equation $x(x + 4) = 45$ in this way gives:

$$x^2 + 4x = 45$$

$$x^2 + 4x - 45 = 0$$

How do you think we could solve this equation?

Solving quadratic equations

We can solve the quadratic equation $x^2 + 4x - 45 = 0$ by factoring the expression on the left-hand side:

$$(x + 9)(x - 5) = 0$$

When two numbers multiply together to make 0, one of the numbers must be 0, therefore we can conclude that either

$x + 9 = 0$ or $x - 5 = 0$

This gives us two solutions that solve the quadratic equation:

$x = -9$ **cm** or $x = 5$ **cm**

In the context of the height of a painting, length cannot be negative. $x = 5$ **cm** is the only valid solution.

However, there are many problems for which both solutions are valid.

Solving quadratic equations

Solve the equation $x^2 = 3x$ by factoring.

Start by rearranging the equation so that the terms are on the left-hand side:

$$x^2 - 3x = 0$$

Factoring the left-hand side gives us:

$$x(x - 3) = 0$$

So $\mathbf{x = 0}$ or $x - 3 = 0$

$\mathbf{x = 3}$

Solve the equation $x^2 - 5x = -4$ by factoring.

Start by rearranging the equation so that the terms are on the left-hand side.

$$x^2 - 5x + 4 = 0$$

We need to find two integers that add together to make –5 and multiply together to make 4.

These two numbers are –1 and –4.

Factoring the left-hand side gives us:

$$(x - 1)(x - 4) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x - 4 = 0$$

$$\mathbf{x = 1} \qquad\qquad \mathbf{x = 4}$$

Solving quadratics by factoring

Question 1/5: Can you solve $x^2 = 3x$ by factoring?

Press the "=" button to show the work step by step.

- $x = 0$ or $x = 3$
- $x = 1$ or $x = 3$
- $x = 0$
- $x = -1$ or $x = -3$

Solving quadratics by factoring

Find x for $x^2 - 7x + 10 = 0$.

$(x + 1)(x + 1) = 0$

1 2 3