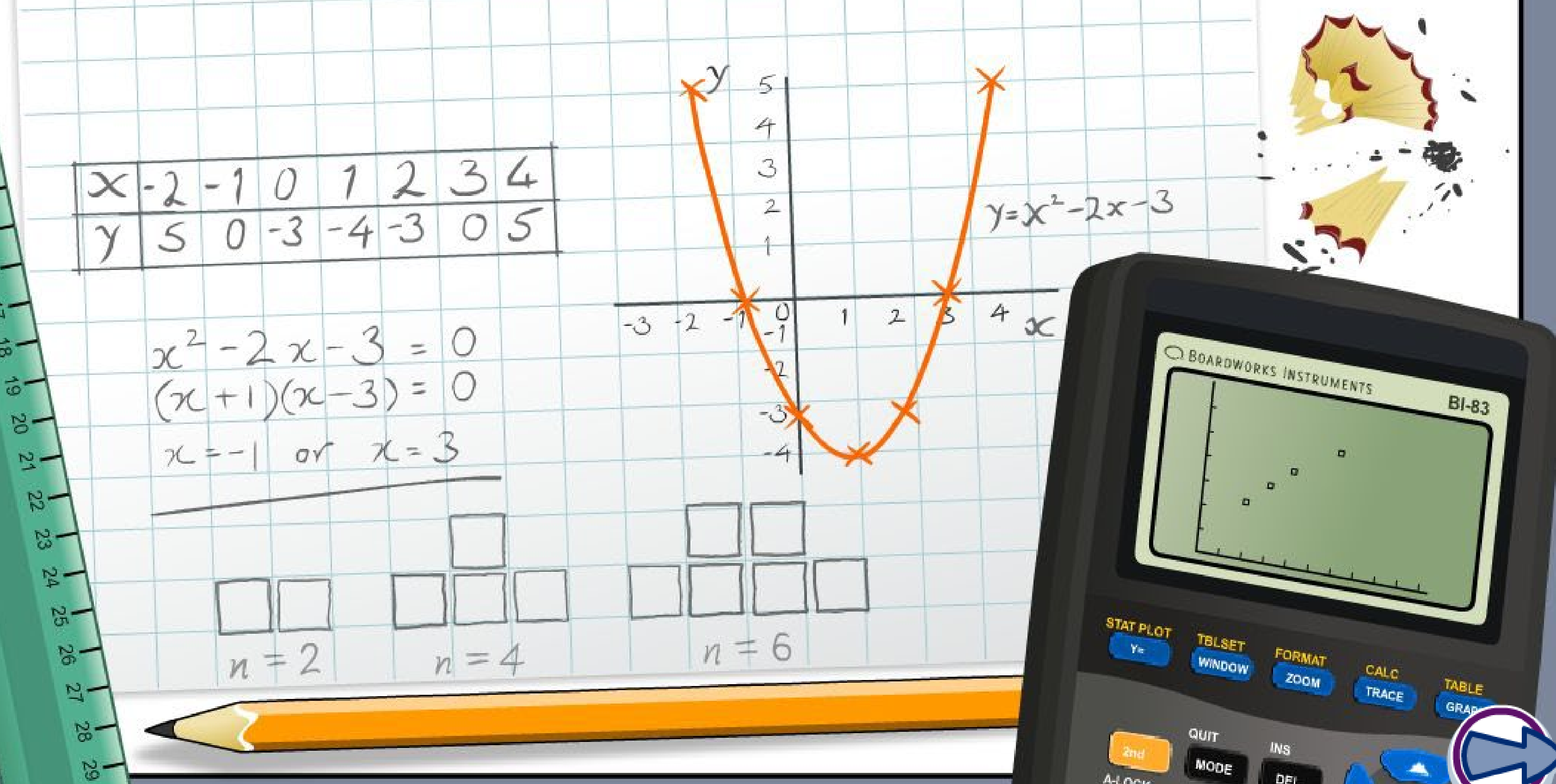
Exponential functions



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Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

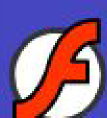
4) Model with mathematics.

5) Use appropriate tools strategically.

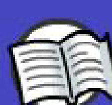
6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.



This icon indicates that the slide contains activities created in Flash. These activities are not editable.



This icon indicates teacher's notes in the Notes field.

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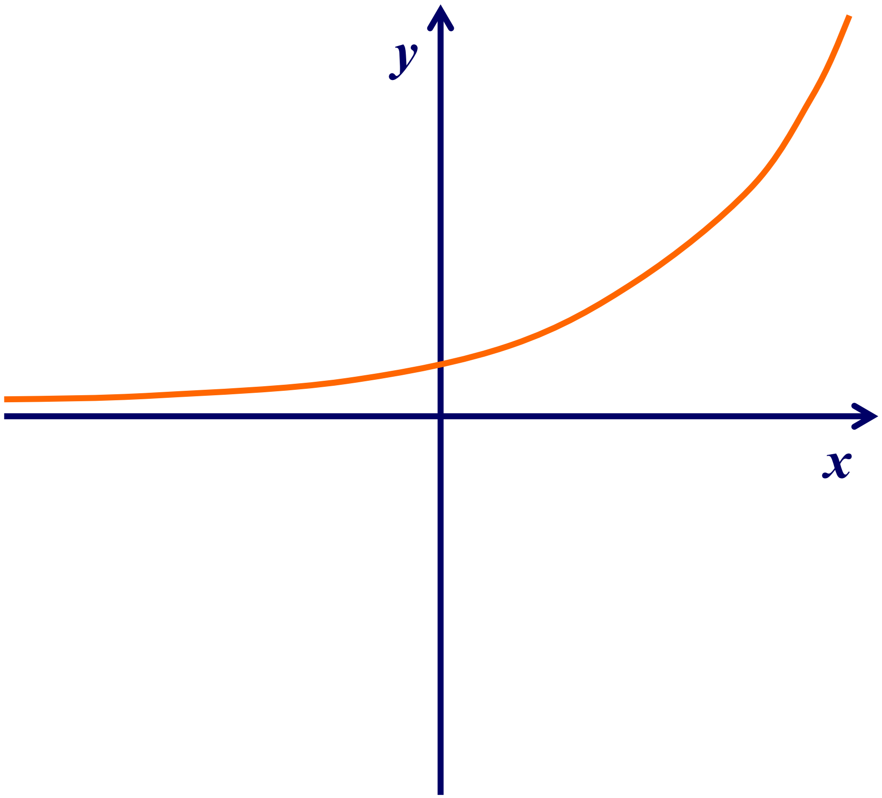
What can you say about this function?



Is it increasing or decreasing?

Does it have a constant rate?

Does it have any maxima or minima?

How does it behave at either end?

How do you think this function is written?

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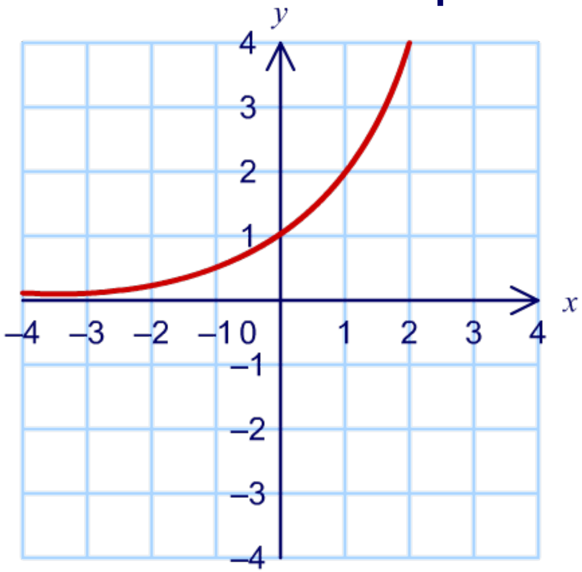
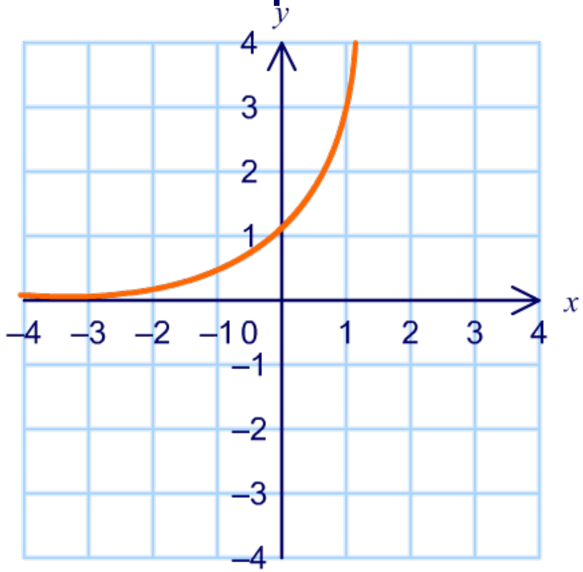
An **exponential function** is a function in the form: $y = a^x$
where a is a positive constant.

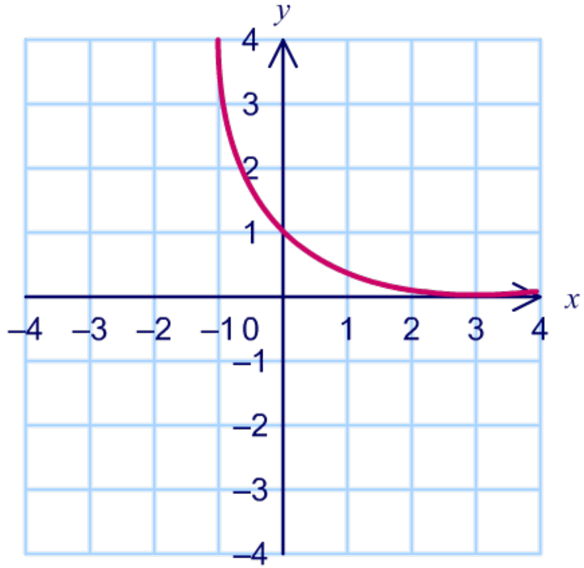
Here are examples of three exponential functions:



$y = 2^x$



$y = 3^x$



$y = 0.25^x$

In each of these examples, the x-axis forms an **asymptote**.

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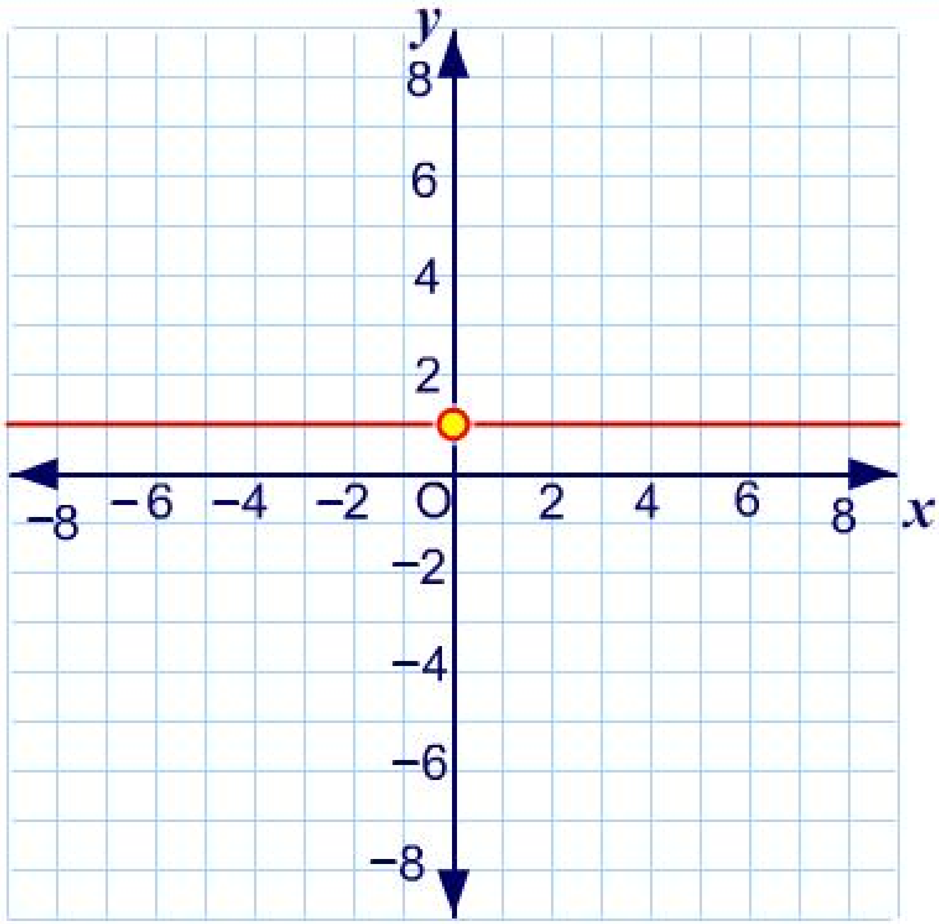
Exploring exponential graphs



x	y
-3	1
-2	1
-1	1
0	1
1	1
2	1

$y = 1^x$

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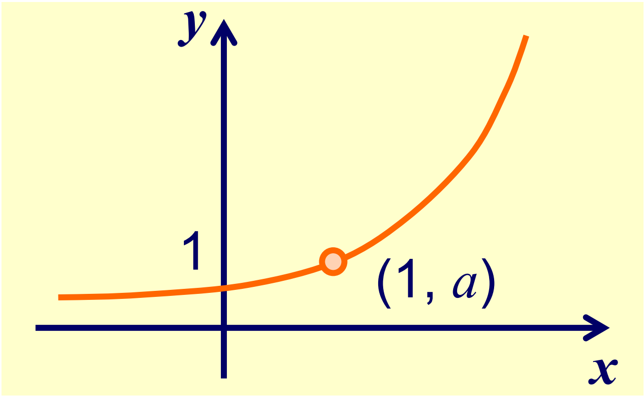
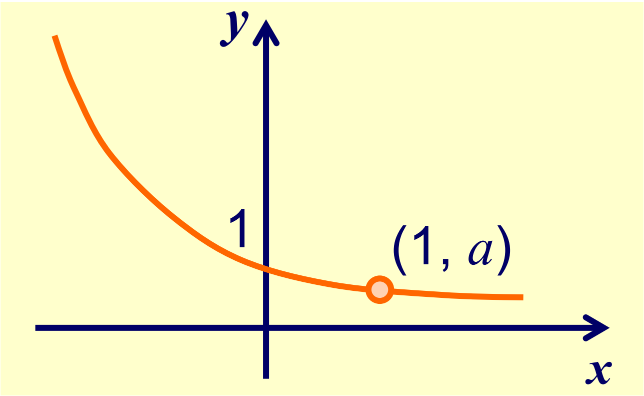
The general form of an exponential function base a is:

$$f(x) = a^x \text{ where } a > 0 \text{ and } a \neq 1$$

When $a > 1$, the graph of $y = a^x$ has the following shape:



When $0 < a < 1$, the graph of $y = a^x$ has the following shape:



In both cases the graph passes through (0, 1) and (1, a).

Why does this happen?

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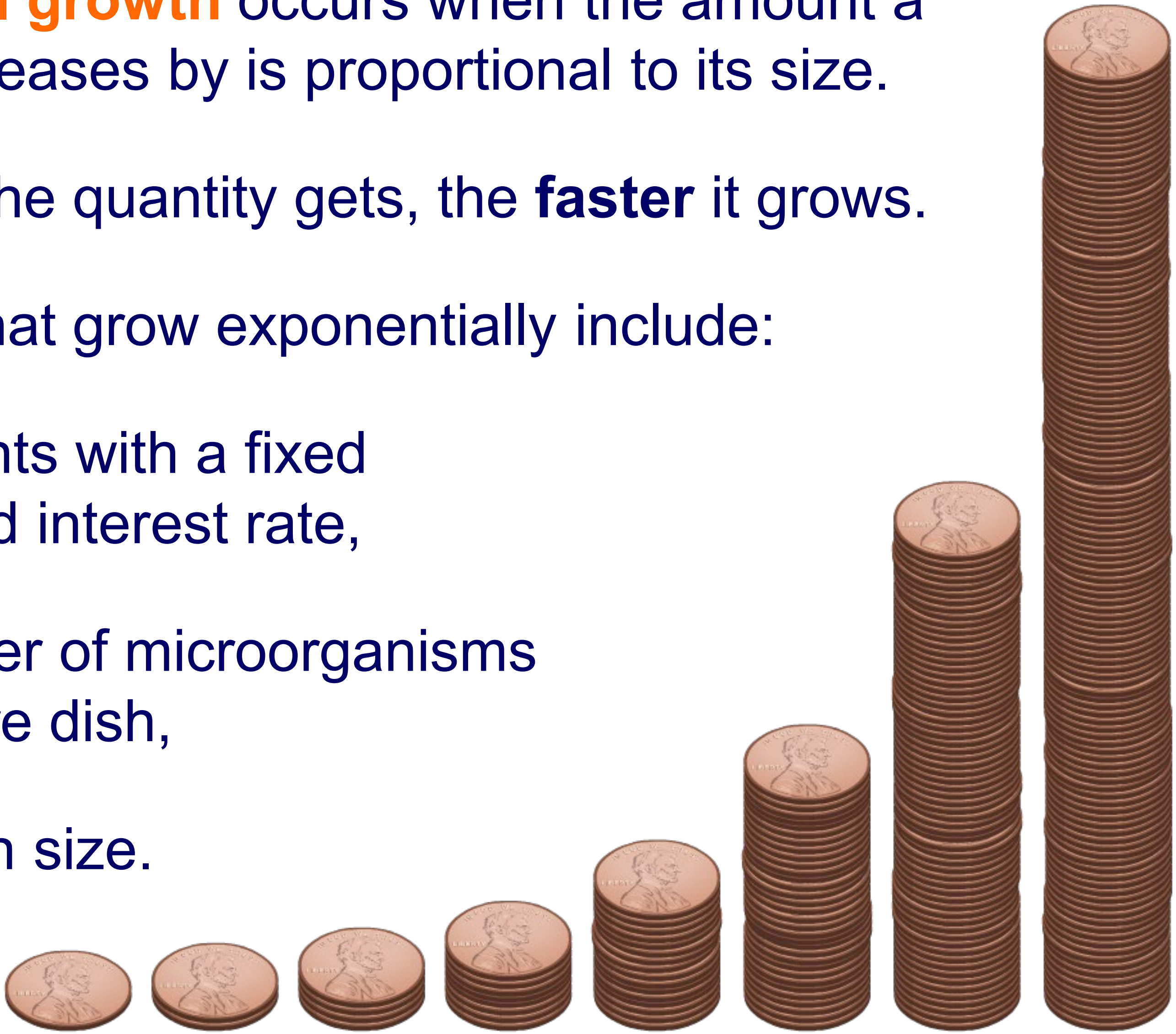
Introducing exponential growth

Exponential growth occurs when the amount a quantity increases by is proportional to its size.

The **larger** the quantity gets, the **faster** it grows.

Quantities that grow exponentially include:

- investments with a fixed compound interest rate,
- the number of microorganisms in a culture dish,
- population size.



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Introducing exponential decay

Exponential decay occurs when the amount a quantity decreases by is proportional to its size.

The **smaller** it becomes, the more **slowly** it decays.

Quantities that decay exponentially include:

- the rate at which an object cools
- the number of atoms in a radioactive isotope
- the value of a car as it **depreciates**.

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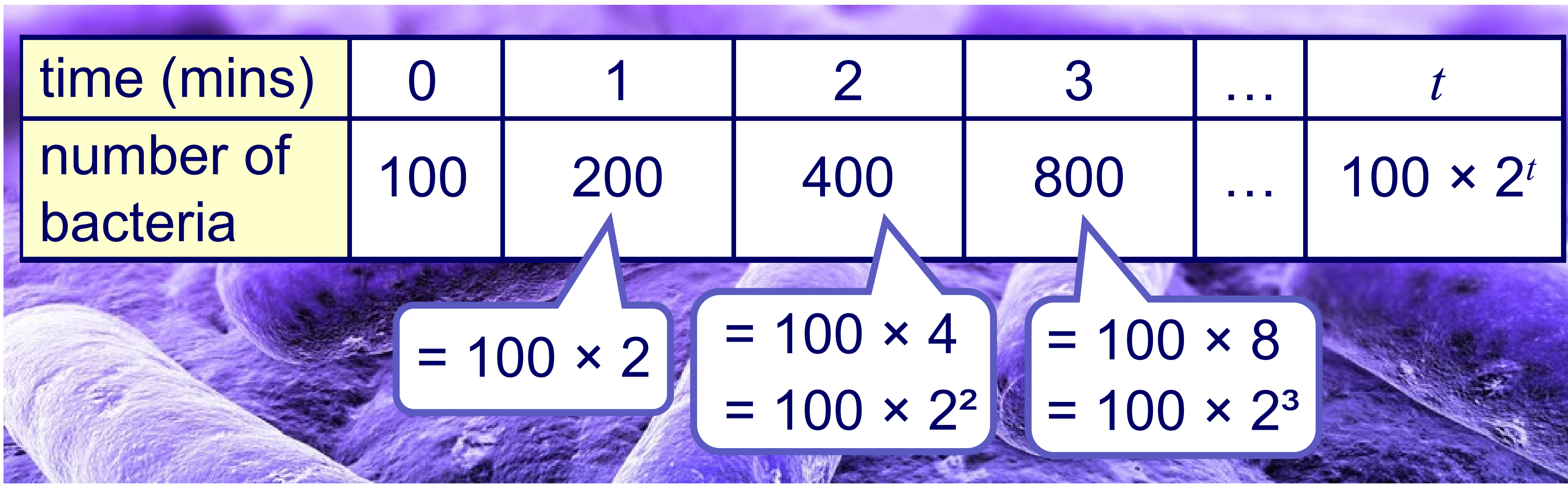
Exponential growth example

A population of 100 bacteria doubles in size every minute. Write a function to model the growth of the bacteria.

We can write the values for the first few minutes in a table:



time (mins)	0	1	2	3	…	t
number of bacteria	100	200	400	800	…	100×2^t

$= 100 \times 2$

$= 100 \times 4$
$= 100 \times 2^2$

$= 100 \times 8$
$= 100 \times 2^3$

If N is the number of bacteria after t minutes, then: $N = 100 \times 2^t$

More generally, if N_0 is the number of bacteria when $t = 0$,

$$\boxed{N = N_0 2^t}$$

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Hayden has been offered two jobs, both with a starting salary of \$20,000. Employer A offers a pay raise of \$500 per year, and employer B promises a 2.4% increase each year.

Where will Hayden have the highest salary after 3 years?
Where will Hayden have the highest salary after 5 years?

Press "**start**" to see the solution.

start

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Compounding interest

You invest $1000 at a 7% annual interest rate for 3 years. Calculate how much money you will have when the account matures if interest is compounded...

a) annually

b) monthly

c) weekly

d) daily

Press on a part of the question to see its solution.

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Compounding interest

If an investment earns more money the more often interest is compounded, I must be able to make the amount as large as I like by compounding sufficiently often!

Press "**play**" to test this claim.

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The number e

This number denoted by e is an irrational number.

$$e = 2.718281828459045235\ldots$$

(to 19 significant digits)

Most scientific calculators have e^x as a secondary function above the key marked “**ln**”.

The function e^x is called the **exponential function** or the **natural exponential**.

This is not to be confused with *an* exponential function, which is any expression of the general form a^x, where a is a constant.

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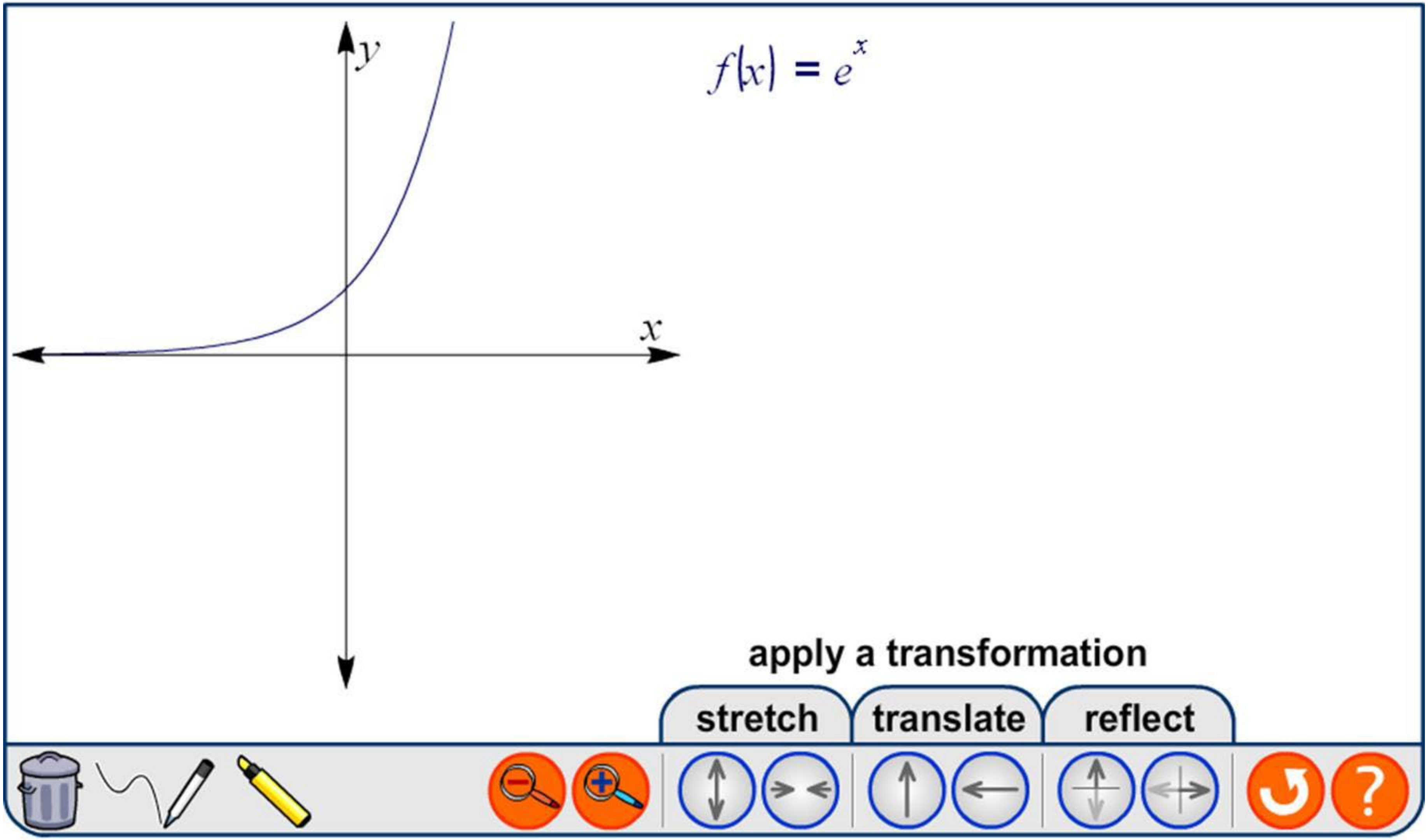
Transformations of $f(x) = e^x$



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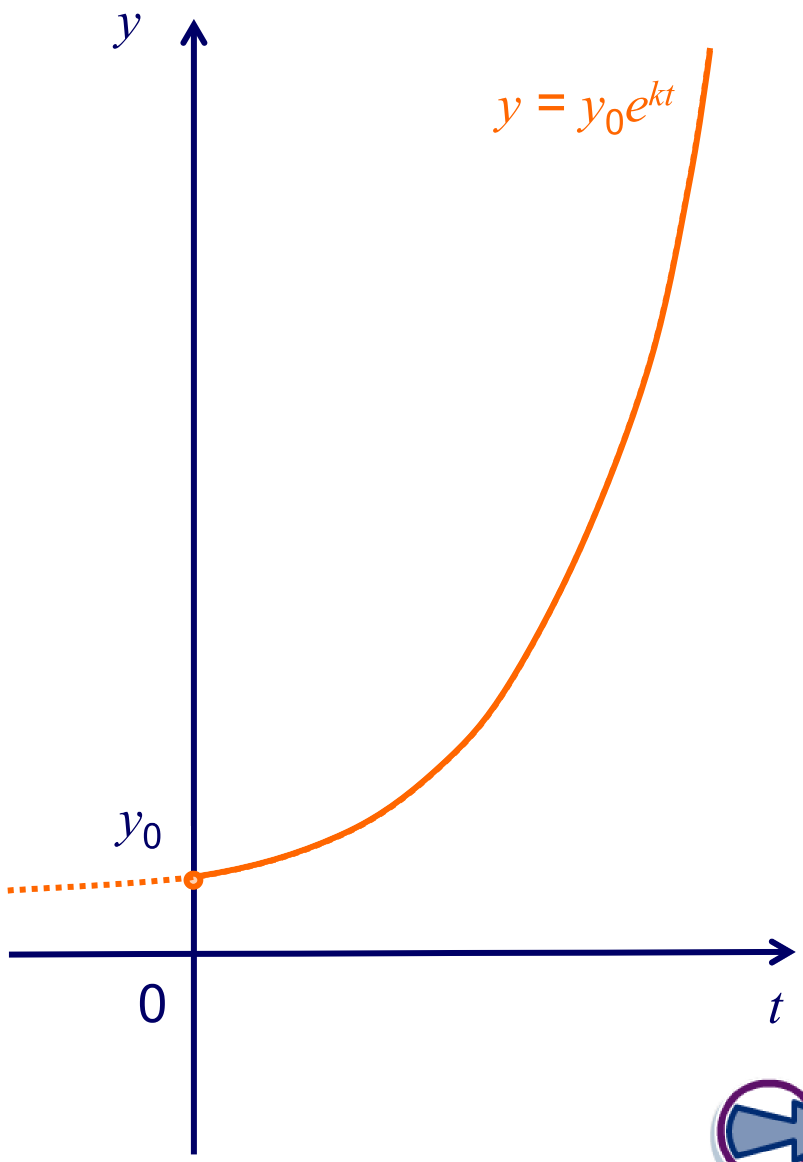
Exponential growth function

In general, exponential growth can be modeled by the function:

$$y = y_0 e^{kt}$$

- t is time
- y_0 is the original quantity (the quantity when $t = 0$)
- y is the quantity after time t
- k is a positive constant (the growth rate)
- the domain is $0 \leq x < \infty$
- the range is $y_0 \leq y < \infty$.



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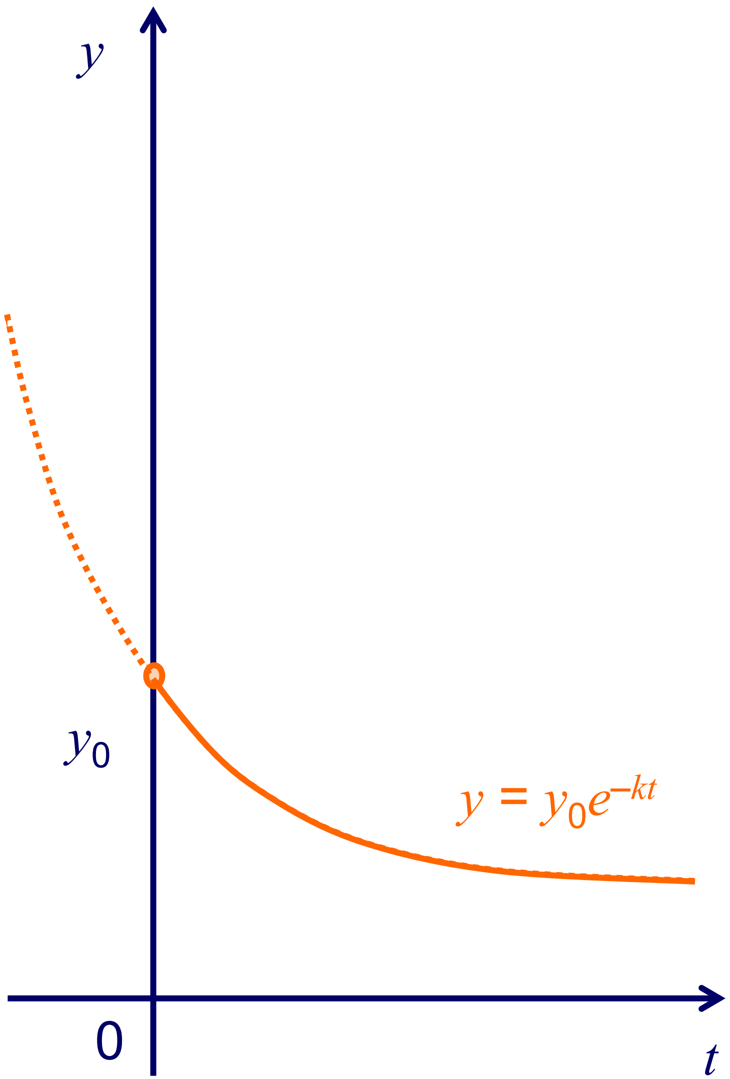
Exponential decay function



In general, exponential decay can be modeled by the function:

$$y = y_0e^{-kt}$$

- t is time
- y_0 is the original quantity (the quantity when $t = 0$)
- y is the quantity after time t
- k is a positive constant (the decay rate)
- the domain is $0 \leq x < \infty$
- the range is $0 < y \leq y_0$.

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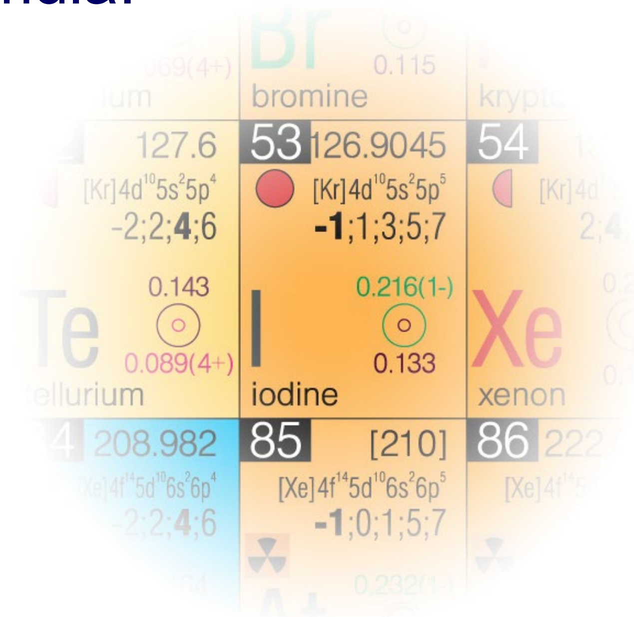
Exponential decay example

The mass of a sample of radioactive iodine, *m* grams, decays according to the formula:

$$m = 15e^{-0.083t}$$



where *t* is the number of days after it is first observed.

a) What is the initial mass of the sample?

b) Sketch the graph of m against t.

c) What is the mass of the sample after 2 days?

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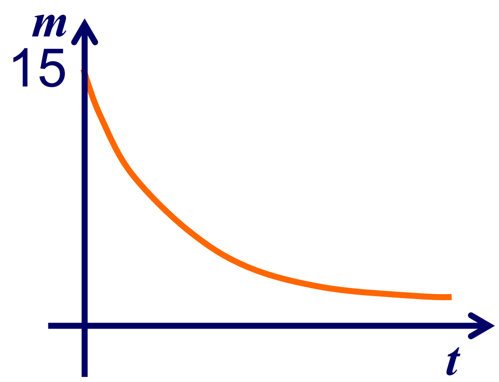
Exponential decay solutions

a) What is the initial mass of the sample?

When $t = 0$, $m = 15e^0$

$= 15$ The initial mass of the sample is **15 g**.

b) Sketch the graph of m against t.



The graph of $m = 15e^{-0.083t}$ will be an exponential decay curve passing through the point (0, 15).

c) What is the mass of the sample after 2 days?

When $t = 2$, $m = 15e^{-0.083 \times 2}$

$= 12.71$ (to the nearest hundredth)

The mass of the sample after 2 days is **12.71 g**.

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