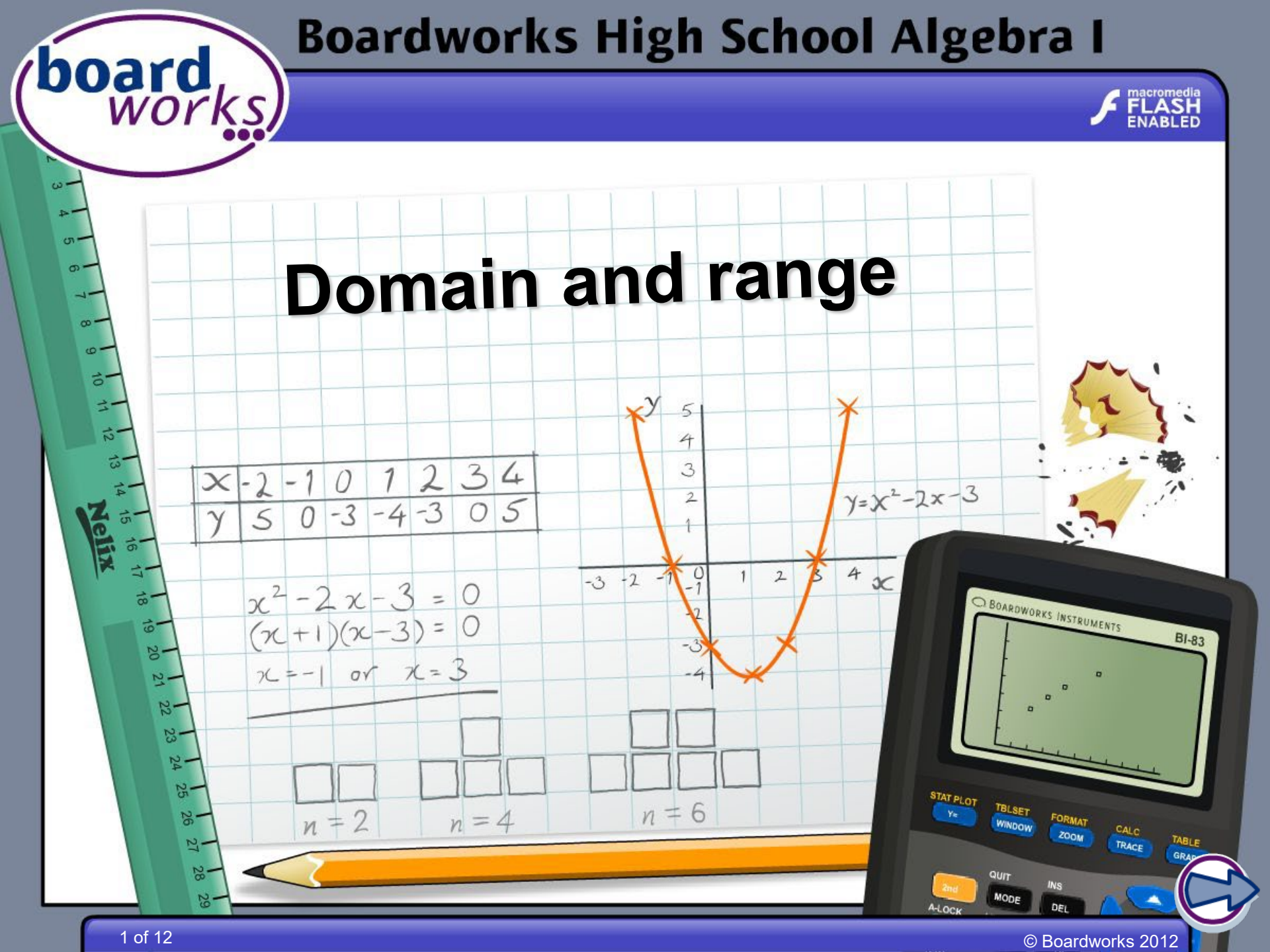
Domain and range



Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

Relations and functions

A **relation** maps values from a starting set, called the **domain**, to a second set called the **range**.

When each value in the domain only maps to **exactly one** value in the range, the relation is a **function**.

A function is only fully defined if we are given:

- the domain (x) of the function, e.g. the set {1, 2, 3, 4}.
- the rule that defines the function, for example $f(x) = x + 2$

Can you state the range of this function?

The domain and range of a function

Given the rule $f(x) = x + 2$ and the domain {1, 2, 3, 4} we can find the range: {3, 4, 5, 6}

A function is defined by $f(x) = 8 - x$ and has the domain {7, 8, 9, 10}. What is the range of this function?

To help, make a table of the x-values (domain) and corresponding $f(x)$-values (range):

x	$8 - x$	$f(x)$
7	8 – 7	1
8	8 – 8	0
9	8 – 9	–1
10	8 – 10	–2

So the range is **{–2, –1, 0, 1}**.

Can you write this function as a set of ordered pairs?

Discrete and continuous

Discrete values | Continuous interval

It is more common for a function to be defined over a **continuous** interval, rather than a set of **discrete** values.

Press on each of the tabs above to see a reminder about these types of values.

The domain and range of a function

The function $f(x) = 4x - 7$ is defined over the domain $-2 \leq x < 5$. Find the range of this function.

Since this is a linear function, the smallest and largest values of x will give the upper and lower bounds of the range.

Solve for the smallest and largest values of x:

When $x = -2$, $f(x) = -8 - 7 = -15$

When $x = 5$, $f(x) = 20 - 7 = 13$

The range of the function is therefore:

$$-15 \leq f(x) < 13$$

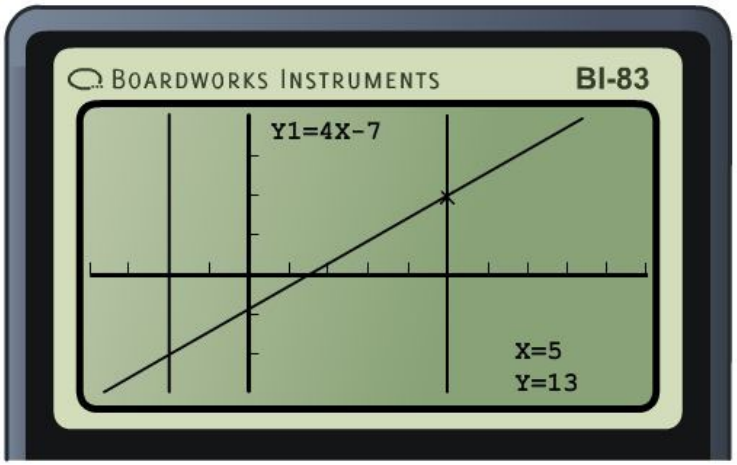
Using your graphing calculator

We can use a graphing calculator to plot a function and find its domain and range.

How do you think we could use the 'intersection' feature of our calculator to find the range of the function $f(x) = 4x - 7$ with domain $-2 \leq x < 5$?

Use the "GRAPH" feature to plot the function and then plot a line at each endpoint of the domain, i.e. $x = -2$ and $x = 5$.

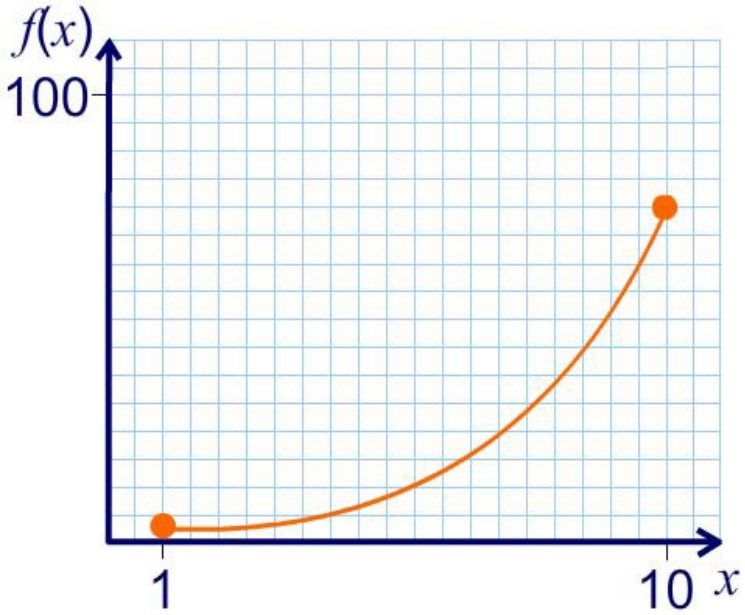
Use the 'intersection' feature to find where these lines cross the function. This will give the endpoints of the range.



$$-15 \leq f(x) < 13$$

Domain and range from graphs

The graph below shows the function $f(x) = x^2 - 3x + 5$ over a restricted domain.



Using the graph, determine the domain and range of $f(x)$.

Notice that the graph stops at $x = 1$ and $x = 10$, and is **continuous** between these points.

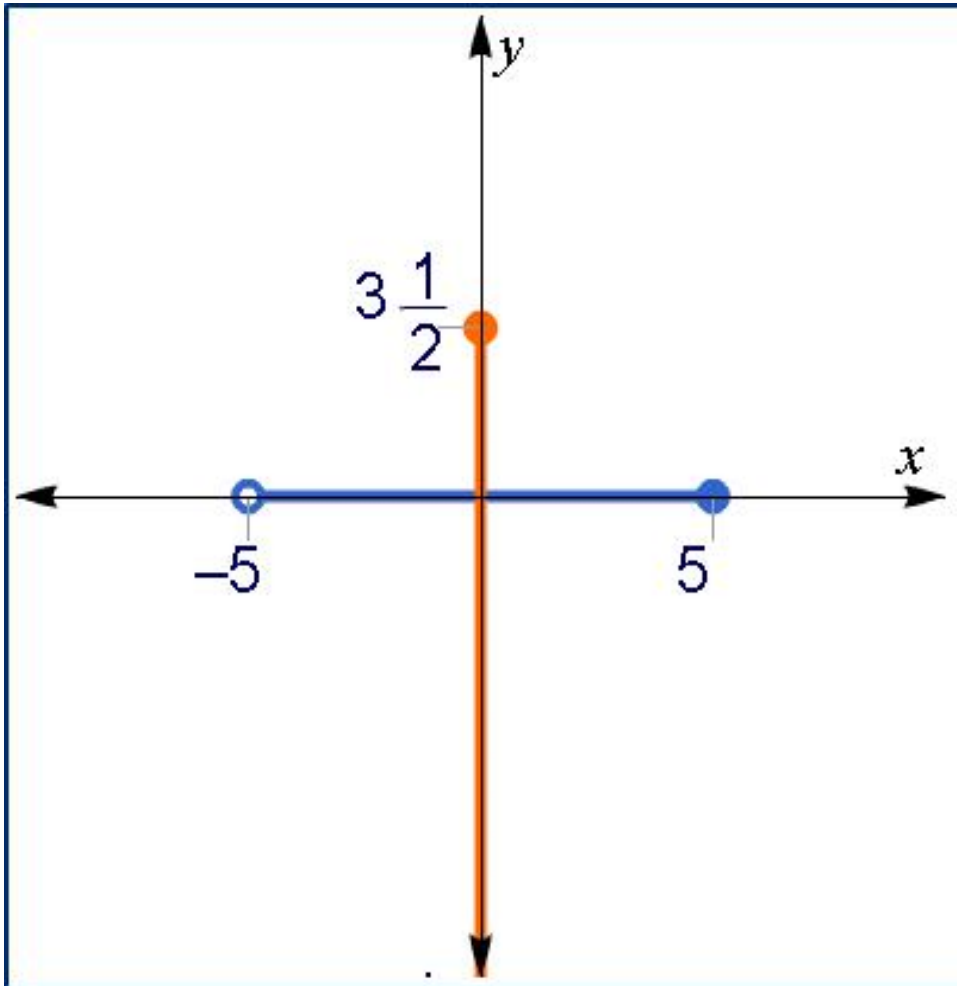
So, the domain is: $1 \leq x \leq 10$

When $x = 1$, $f(x) = 3$, and when $x = 10$, $f(x) = 75$.

The range of the function is therefore: $3 \leq f(x) \leq 75$

If a new condition states that $f(x)$ has to be less than 45, what is the new domain of the function?

How domain affects range



$$f(x) = \frac{3x}{2} - 4$$

$$-5 < x \leq 5$$

Appropriate domain/range

State an appropriate domain and range for...

a) ...the function that gives the number of spectators, s, needed to fill n seats in a baseball stadium with at least 350 seats.

Press on each question to see the answer.

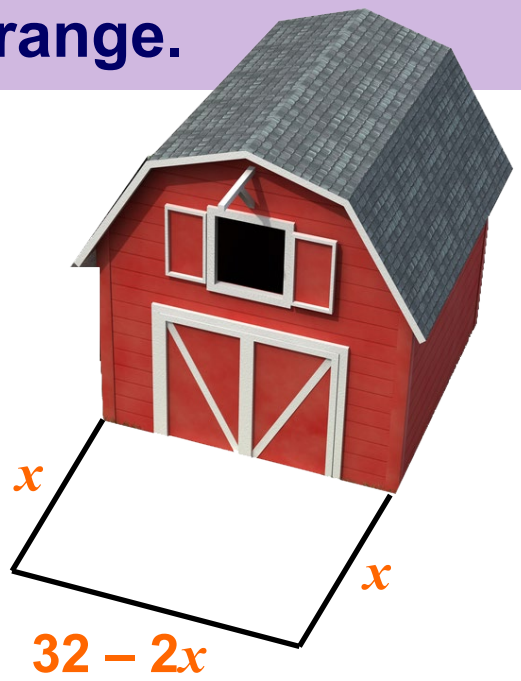
b) ...the function that gives the number of hours, h, it takes a factory to make n cars.

c) ...the function describing the number of 2 liter containers (c) that l liters of water will fill, if you have 12 liters.

Dog's play area

Andy's dad wants to create a rectangular area for his dog to play in. He plans to use one side of his barn and 32 feet of fencing to enclose the play area, as shown in the diagram.

Find a function to model the largest possible area for the dog to play in. State its domain and range.



Let x be the length of the play area.

The width of the area will be: $32 - 2x$ ft.

A function for the area is therefore:
$A(x) = x(32 - 2x)$

Since the dimensions of the area cannot be negative, think: $x \geq 0$ and $32 - 2x \geq 0$.

Solving, we see that the domain is: $\mathbf{0 \leq x \leq 16}$.

Graphing the function

The area function is $A(x) = x(32 - 2x)$, with domain $0 \le x \le 16$.

When $x = 0$ we get $A(0) = 0$. When $x = 16$ we get $A(16) = 0$.

We need to investigate further to see the range of this function.

Use your graphing calculator to investigate the function and find the range.

Graph the function and use the maximum feature to find the upper bound of the range.

The range is **$0 \le A(x) \le 128$**.

The largest area that Andy's dog will have to run around in is **128 square feet**.