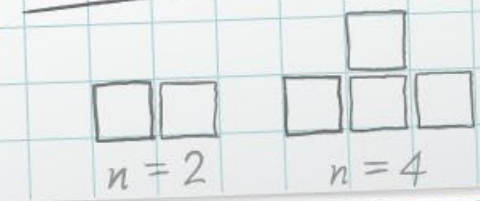
Arithmetic sequences

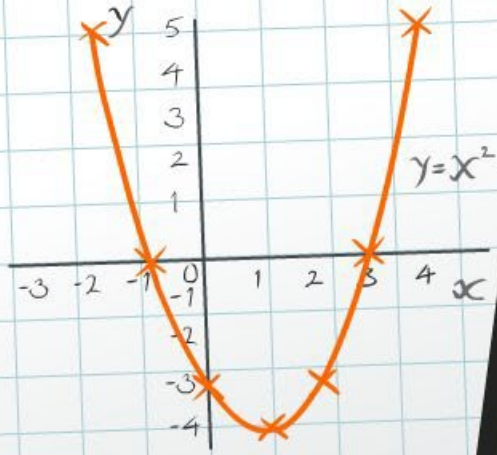
x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

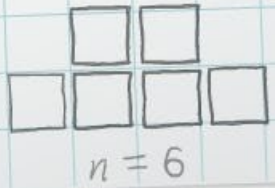
$x^2 - 2x - 3 = 0$

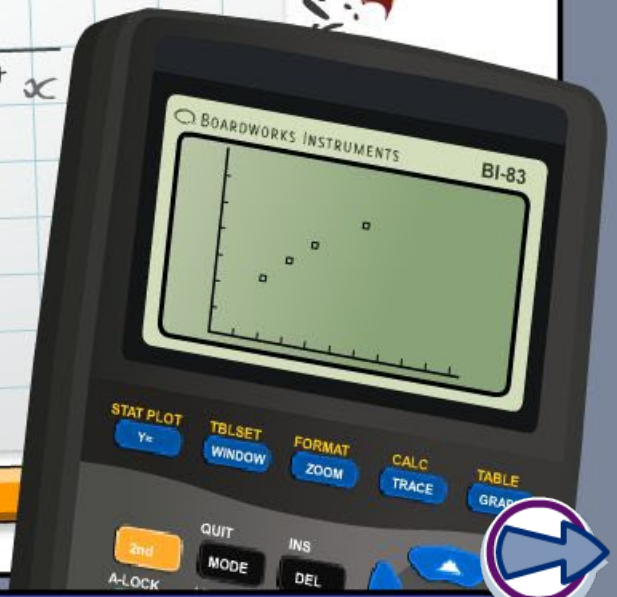
$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$









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Information

Common core icons

This icon indicates a slide where the Standards for Mathematical Practice are being developed. Details of these are given in the Notes field.

Slides containing examples of mathematical modeling are marked with this stamp.

This icon indicates an opportunity for discussion or group work.

The **Standards for Mathematical Practice** outlined in the Common Core State Standards for Mathematics describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

They are:

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

This icon indicates that the slide contains activities created in Flash. These activities are not editable.

This icon indicates teacher's notes in the Notes field.

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Arithmetic sequences

What do these sequences have in common?

3, 6, 9, 12, 15 …

add 3

5, 7, 9, 11, 13 …

add 2

2, 7, 12, 17, 22…

add 5

8, 11, 14, 17, 20, …

add 3

100, 104, 108, 112, 116…

add 4

All these sequences are made by **adding** on the same number each time. They are **arithmetic sequences**.

The difference between any two consecutive terms in an arithmetic sequence is called the **common difference**, d.

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Recursive definition

Each term in a sequence is identified by its position in the sequence.

The first term is a_1, because it is in position 1.

The term in position n, where n is a natural number, is called a_n.

An arithmetic sequence can be defined **recursively** by the formula:

$$a_{n+1} = a_n + d$$

The value of the first term needs to be given as well, so that the definition is unique.

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The general term

How do you find the **general term** of an arithmetic sequence?

8, 11, 14, 17, 20…

+ 3 + 3 + 3 + 3

This is an arithmetic sequence with 3 as the common difference. We could write this sequence as:

$8,\ \ 8 + 3,\ \ 8 + 3 + 3,\ \ 8 + 3 + 3 + 3,\ \ldots\ ,\ 8 + \underbrace{3 + \ldots + 3}_{n - 1 \text{ times}},\ \ldots$

or

$8,\ \ 8 + 3,\ \ 8 + (2 \times 3),\ \ 8 + (3 \times 3),\ \ldots\ \ ,\ 8 + ((n - 1) \times 3),\ \ldots$

The general term of this sequence is **$8 + 3(n - 1)$**

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Plotting terms

Arithmetic sequence and linear functions

An arithmetic sequence is a linear function on the natural numbers.

Press "**play**" to see how this can help to find an explicit formula for the general term of a sequence.

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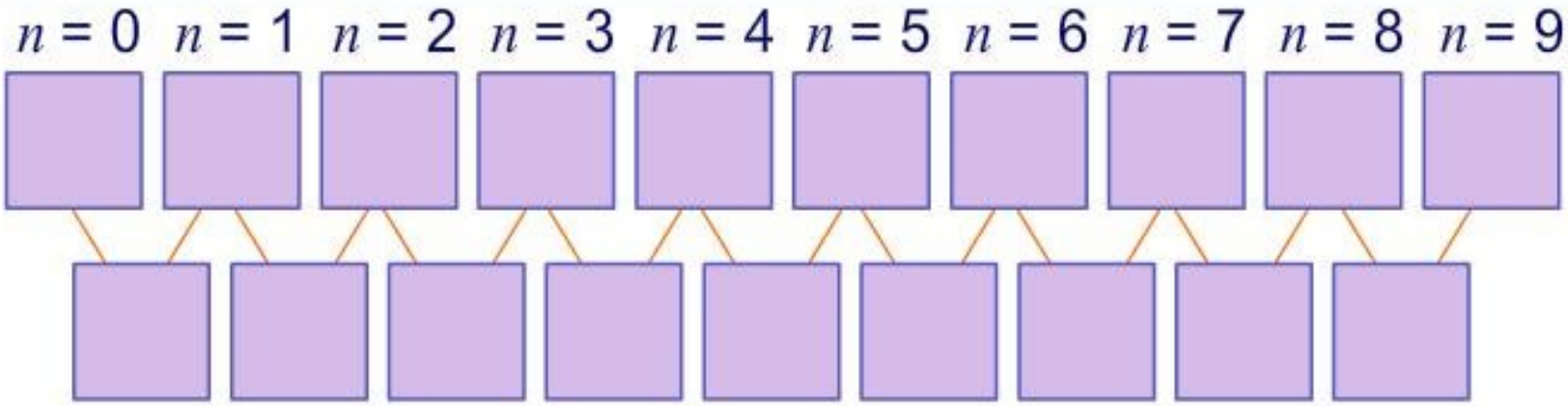
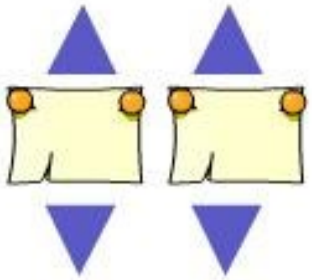
Investigating arithmetic sequences





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Explicit formula

If we call the first term of an arithmetic sequence a_1 and the common difference d we can write a general arithmetic sequence as:

1st	2nd	3rd	4th	5th
a_1,	$a_1 + d$,	$a_1 + 2d$,	$a_1 + 3d$,	$a_1 + 4d$, …
	+ d	+ d	+ d	+ d

The **explicit formula** for an arithmetic sequence with common difference d is given by

$$a_n = a_1 + (n - 1)d$$

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Deriving the general equation

We can derive the **general form** of the explicit formula for an arithmetic sequence with common difference d from the equation of a line.

The point–slope form of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

Write n for x, a_n for y, and d for m:

$$a_n - y_1 = d(n - x_1)$$

Use the first term in the sequence, so $x_1 = 1$ and $y_1 = a_1$:

$$a_n - a_1 = d(n - 1)$$

Rearrange to isolate a_n:

$$\boldsymbol{a_n = a_1 + d(n - 1)}$$

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Finding the n^{th} term in arithmetic sequences

Question 1/5: What is the n^{th} term of the arithmetic sequence 7, 9, 11, 13, 15, ...?

Press the "=" button to show the calculations step by step.

- $a_n = 2n$
- $a_n = 2n + 7$
- $a_n = 2n + 5$
- $a_n = 5n + 2$

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Comparing definitions

A sequence can be represented by its first terms, a recursive definition, an explicit formula, or a graph of the terms. Can you recognize when these represent the same sequence? Select the sequence that does not match the others.

Press **start** to begin.

start

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Car hire problems

The cost of hiring a car from Carrie's Cars depends on the number of days the car is needed for:

Days	1	2	3	4	5
Cost	$60	$80	$100	$120	$140

Find the explicit formula and use it to solve these problems:

1) What is the cost for 6 days?

2) How much will it cost for 2 weeks car hire?

3) How many days can Lois hire a car for if she has $250?

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Car hire solutions

The car hire cost is an arithmetic sequence with common difference 20.

60, 80, 100, 120, 140, …

Find the explicit formula first:

$a_n = a + (n - 1)d = 60 + (n - 1)20 =$ **$40 + 20n$**

1) The cost for 6 days is the 6th term:
   140 + 20 = **$160**.

2) For 2 weeks, n = 14 so it will cost **$320**.

3) Lois has $250, so solve $250 = 40 + 20n$ to get $n = 10.5$.
   Lois can only hire the car for full days so n must be an integer less than 10.5. She can hire the car for **10 days**.

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Theater trip problem

MODELING

A group of 34 school students go on a trip to the theater, accompanied by 2 teachers. There are 8 seats in the front row of the theater, and every row behind that has 2 extra seats. The school group take up 3 consecutive rows.

Which rows are they sitting in?

press here to see the method

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